

Glossary: Mathematical Terms, Tables, and Illustrations

This Glossary contains terms found in the 2017 *Massachusetts Curriculum Framework for Mathematics*, as well as selected additional terms.

A

Absolute value. The absolute value of a real number is its (non-negative) distance from 0 on a number line.

Addition and subtraction within 5, 10, 20, 100, or 1,000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively.

Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. *Example: $3/4$ and $-3/4$ are additive inverses of one another because $3/4 + (-3/4) = (-3/4) + 3/4 = 0$.*

Algorithm/Standard Algorithm:

Algorithm. A finite set of steps for completing a procedure, e.g., multi-digit operations (addition, subtraction, multiplication, division). (*See standard 3.NBT.2.*)

Standard algorithm. One of the conventional algorithms used in the United States is based on place value and properties of operations for addition, subtraction, multiplication, and division. (*See standards 4.NBT.4, 5.NBT.5, and 6.NS.2.*) [See Table 3 in the Glossary.](#)

Analog. Having to do with data represented by continuous variables, e.g., a clock with hour, minute, and second hands. (M)

Associative property of addition. (*See Table 3 in the Glossary.*)

Associative property of multiplication. [See Table 3 in the Glossary.](#)

Attribute. A common feature of a set of figures.

Array. An arrangement of objects in rows and columns (often appearing as a grid) is frequently used to represent multiplication.

B

bivariate data. Pairs of linked numerical observations. *Example: a list of heights and weights for each player on a football team.*

Box plot. A graphic method that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data. (DPI)

C

Capacity. The maximum amount or number that can be contained or accommodated, e.g., a jug with a one-

gallon *capacity*; the auditorium was filled to *capacity*.

Cardinal number. A number (such as 1, 5, 15) that is used in simple counting and indicates how many elements there are in a set.

Coefficient. Any of the factors of a product considered in relation to a specific factor. (W)

Commutative property. See Table 3 in the Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Compose numbers. a) Given pairs, triples, etc. of numbers, identify sums or products that can be computed; b) Each place in the base-ten place value is composed of ten units of the place to the left, i.e., one hundred is composed of ten bundles of ten, one ten is composed of ten ones, etc.

Compose shapes. Join geometric shapes without overlaps to form new shapes.

Composite number. A whole number that has more than two factors. (H)

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: **algorithm**; **computation strategy**.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order and may be aimed at converting one problem into another. See also: **computation algorithm**.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Coordinate plane. A plane in which a point is represented using two coordinates that determine the precise location of the point. In the Cartesian plane, two perpendicular number lines are used to determine the locations of points. In the polar coordinate plane, points are determined by their distance along a ray through that point and the origin, and the angle that ray makes with a pre-determined horizontal axis. (OK)

Counting number. A number used in counting objects, i.e., a number from the set 1, 2, 3, 4, 5, See Illustration 1 in the Glossary.

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have eight books and three more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

D

Data display. A representation used as a visual summarization of a set of data, such as tables, bar graphs, histograms, circle graphs, box plots, line plots (dot plots), line graphs, and scatter plots.

Data set. A collection of numbers or objects that relate to a subject of interest.

Decimal expansion. Writing a rational number as a decimal.

Decimal fraction. A fraction (as $0.25 = \frac{25}{100}$ or $0.025 = \frac{25}{1000}$) or mixed number (as $3.025 = 3 \frac{25}{1000}$) in which the denominator is a power of ten, usually expressed by the use of the decimal point. (M)

Decimal number. Any real number expressed in base ten notation, such as 2.673.

Decompose numbers. Given a number, identify pairs, triples, etc. of numbers that combine to form the given number using subtraction and division.

Differences between parameters. A difference of numerical characteristics of a population, including measures of center and/or spread.

Digit. a) Any of the Arabic numerals 1 to 9 and usually the symbol 0; b) One of the elements that combine to form numbers in a system other than the decimal system.

Digital. Having to do with data that is represented in the form of numerical digits; providing a readout in numerical digits, e.g., a digital watch.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center and multiplies distances from the center by a common scale factor.

Directrix. A parabola is the collection of all points in the plane that are the same distance from a fixed point, called the focus (F), as they are from a fixed line, called the directrix (D). (Lone Star College lonetsar.edu)

Divisor. A number by which another number is divided.

Dot plot. See: *line plot*.

E

Equation. A mathematical expression that equates two quantities (must have an equal sign.)

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Exponent. The number that indicates how many times the base is used as a factor, e.g., in $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times.

Exponential function. A function of the form $y = a \bullet b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$. The variables do not have to be x and y . For example, $A = 3.2 \bullet (1.02)^t$ is an exponential function.

Expression. A mathematical phrase that combines numbers, variables, exponents, radicals, and/or operators (e.g., $a2 \div 3$); may or may not have an equal sign.

F

Factor. Each of two or more numbers or expressions that are multiplied to result in a product.

Fibonacci sequence. The sequence of numbers beginning with 1, 1, in which each number that follows is the

sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144³².

First quartile. For a data set with median M , the first quartile is the median of the data values less than M . Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.32 See also: median, third quartile, interquartile range.

Fluency. Fluency in the grades 1–6 standards is the ability *to carry out calculations* and apply *numerical algorithms* quickly and accurately. Fluency in each grade involves a mixture of knowing some answers from memory (instant recall), knowing some answers from patterns (e.g., “adding 0 yields the same number”), and knowing some answers from the use of other strategies. The development of fluency follows a specific progression in these grades that begins with conceptual understanding and eventually requires students to “know from memory their math facts,” use various strategies to arrive at answers, and develop proficiency using the standard algorithm for each operation. (See standards 1.OA.3, 2.OA.2, 3.OA.5, 3.OA.7 and 3.NBT.2, 4.NBT.4, 5.NBT.5, 6.NS.2 and 6.NS.3.)

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a nonnegative number.) See also: **rational number**.

Function. A mathematical relation for which each element of the domain corresponds to exactly one element of the range. (MW)

Function notation. A notation that describes a function. For a function f , when x is a member of the domain, the symbol $f(x)$ denotes the corresponding member of the range (e.g., $f(x) = x + 3$).

G

H

Histogram. A type of bar graph is used to display the distribution of measurement data across a continuous range.

I

Identity property of 0. See Table 3 in the Glossary.

Input-output table. A table that shows the relationship between two or more sets of numbers, each of which follows a mathematical rule.

Integer. All positive and negative whole numbers, including zero. (MW)

Interquartile range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is $15 - 6 = 9$. See also: **first quartile**, **third quartile**.

Irrational number. A number that cannot be expressed as a quotient of two integers, e.g., $\sqrt{2}$. It can be shown that a number is irrational if and only if it cannot be written as a repeating or terminating decimal.

³² Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

K

Know from Memory. To instantly recall single-digit math facts to use when needed. *Note: In the early grades,*

students develop number sense and fluency in operations. Students are expected to commit single digit math facts to memory by the end of: a) grade 2 for addition and related subtraction facts (see standard 2.OA.2); and b) grade 3 for multiplication and related division facts (see standard 3.OA.7).

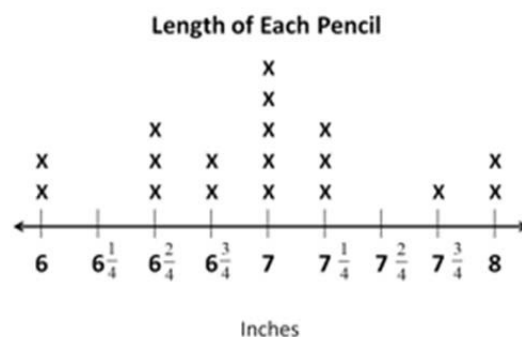
L

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. (Also known as a *dot plot*.) (DPI)

Linear association. Two variables have a linear association if a scatter plot of the data can be well approximated by a line.

Linear equation. Any equation that can be written in the form $Ax + By + C = 0$ where A and B cannot both be 0. The graph of such an equation is a line.

Linear function. A function with an equation of the form $y = mx + b$, where m and b are constants



M

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.³³ *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.*

Mean absolute deviation. A measure of variation in a set of numerical data, is computed by adding the distances between each data value and the mean, then dividing by the number of data values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.*

Measure of variability. A determination of how much the performance of a group deviates from the mean or median, most frequently used measure is standard deviation.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list; or the mean of the two central values, if the list contains an even number of values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.*

Mode. The value (or values) in a data set which appears most frequently.

Model. A mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real-world or mathematical objects, properties, actions, or relationships. (DPI)

Multiple. One of a set of numbers that are products of a specific factor and all other numbers.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0–100. *Example: $72 \div 8 = 9$.*

N

Non-linear association. The relationship between two variables is nonlinear if the change in the second is not simply proportional to the change in the first, independent of the value of the first variable.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Numeral. A symbol or mark used to represent a number.

O

Observational study. A type of study in which an action or behavior is observed in such a manner that no interference with, or influence upon, the behavior occurs.

Order of Operations. Convention adopted to perform mathematical operations in a consistent order. 1. Perform all operations inside parentheses, brackets, and/or above and below a fraction bar in the order specified in steps 3 and 4; 2. Find the value of any powers or roots; 3. Multiply and divide from left to right; 4. Add and subtract from left to right. (NCTM)

³³ To be more precise, this defines the *arithmetic mean*.

Ordinal number. A number designating the place (as first, second, or third) occupied by an item in an ordered sequence. (M)

P

Partition. A process of dividing an object into parts.

Pascal's triangle. A triangular arrangement of numbers in which each row starts and ends with 1, and each other number is the sum of the two numbers above it. (H)

Percent rate of change. A rate of change expressed as a percent. *Example: if a population grows from 50 to 55 in a year, it grows by $5/50 = 10\%$ per year.*

Periodic phenomena. Naturally recurring events, for example, ocean tides, and machine cycles.

Picture (Picto) graph. A graph that uses pictures to show and compare information.

Plane. A flat surface that extends infinitely in all directions.

Polynomial function. Any function whose value is the solution of a polynomial.

Postulate. A statement accepted as true without proof.

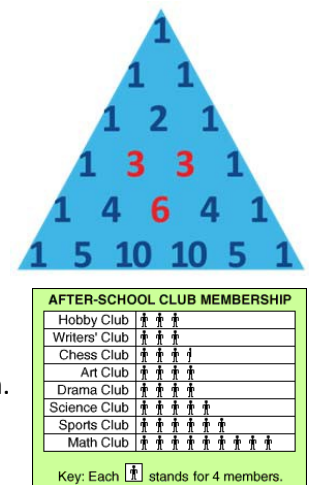
Prime factorization. A number is written as the product of all its prime factors. (H)

Prime number. A whole number greater than 1 whose only factors are 1 and itself.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. [See Table 4 in the Glossary.](#)

Properties of inequality. [See Table 5 in the Glossary.](#)



Properties of operations. [See Table 3 in the Glossary.](#)

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also: uniform probability model.*

Proof. A proof of a mathematical statement is a detailed explanation of how that statement follows logically from statements already accepted as true.

Proportion. An equation that states that two ratios are equivalent, e.g., $4/8 = 1/2$ or $4 : 8 = 1 : 2$.

Pythagorean Theorem. For any right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Q

Quadratic expression. An expression that contains the square of the variable, but no higher power of it.

Quadratic function. A function that can be represented by an equation of the form $y = ax^2 + bx + c$, where a , b , and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of this function is a parabola. (DPI)

Quadratic polynomial. A polynomial where the highest degree of any of its terms is 2.

R

Radical. The $\sqrt{\quad}$ symbol, which is used to indicate square roots or n th roots. (MW)

Random sampling. A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects. (H)

Random variable. An assignment of a numerical value to each outcome in a sample space. (M)

Ratio. A relationship between quantities such that for every a units of one quantity there are b units of the other. A ratio is often denoted by $a:b$ and read “ a to b ”.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b . The rational numbers include the integers. [See Illustration 1 in the Glossary.](#)

Real number. A number from the set of numbers consisting of all rational and all irrational numbers. [See Illustration 1 in the Glossary.](#)

Rectangular array. An arrangement of mathematical elements into rows and columns.

Rectilinear figure. A polygon all angles of which are right angles.

Recursive pattern or sequence. A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.

Reflection. A type of transformation that flips points about a line, is called the *line of reflection*. Taken together, the image and the pre-image have the line of reflection as a line of symmetry.

Relative frequency. The empirical counterpart of probability. If an event occurs N' times in N trials, its relative frequency is $\frac{N'}{N}$. (M)

Relatively Prime. Two positive integers that share no common divisors greater than 1; that is, the only common positive factor of the two numbers is 1.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

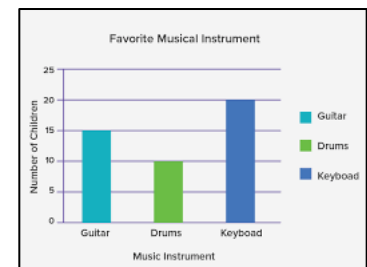
Rotation. A type of transformation that turns a figure into a fixed point, is called the *center of rotation*.

S

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scaled bar graph. A bar graph (or pictograph) that uses intervals of more than one unit.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot. (DPI)



Scientific notation. A widely used floating-point system in which numbers are expressed as products consisting of a number between 1 and 10 multiplied by an appropriate power of 10, e.g., $562 = 5.62 \times 10^2$. (MW)

Sequence, progression. A set of elements is ordered so that they can be labeled with consecutive positive integers starting with 1, e.g., 1, 3, 9, 27, 81. In this sequence, 1 is the *first term*, 3 is the *second term*, 9 is the *third term*, and so on.

Significant figures (digits). A way of describing how precisely a number is written, particularly when the number is a measurement. (MW)

Similarity transformation. A rigid motion followed by a dilation.

Sine (of an acute angle). The trigonometric function for an acute angle is the ratio between the leg opposite the angle when the angle is considered part of a right triangle and the hypotenuse. (M)

Symmetry. Reflection symmetry: a proportional similarity found in two halves of an object (half of the object is the mirror image of the other half, with the division over a line); rotation symmetry: a similarity found in an object over a rotation of fewer than 360 degrees of rotation.

T

Tangent. a) Meeting a curve or surface in a single point if a sufficiently small interval is considered. b) The trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle and the leg adjacent to the angle when the angle is considered part of a right triangle. (MW)

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0. A terminating decimal is the decimal form of a rational number. *See also: repeating decimal.*

Third quartile. For a data set with a median M , the third quartile is the median of the data values greater than M . *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.*

Three-dimensional figure. A (solid) geometric figure consisting of three dimensions, among them length, width, height, and radius.

Transformation. A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the *pre-image*) and the points in another geometric object (the *image*). Reflections, rotations, translations, and dilations are particular examples of transformations.

Translation. A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size. (MW)

Trapezoid. a quadrilateral with at least one pair of parallel sides. (Note: There are two definitions for the term trapezoid. This is the inclusive definition)

Trigonometric function. A function (as the sine, cosine, tangent, cotangent, secant, or cosecant) of an arc or angle is most simply expressed in terms of the ratios of pairs of sides of a right-angled triangle. (M)

Trigonometry. The study of triangles, with emphasis on calculations involving the lengths of sides and the measure of angles. (MW)

Two-dimensional figure. A figure composed of line segments and/or curves that lie on a plane, such as a triangle, circle, quadrilateral (square, trapezoid, rectangle, rhombus, kite), polygon, etc.

U

Uniform probability model. A probability model assigns equal probability to all outcomes. *See also: probability model.*

Unit fraction. A fraction with a numerator of 1, such as $1/3$ or $1/5$.

V

Valid. a) Well-grounded or justifiable; being at once relevant and meaningful, e.g., a *valid* theory; b) Logically correct. (MW)

Variable. A quantity that can change or that may take on different values. Refers to the letter or symbol representing such a quantity in an expression, equation, inequality, or matrix. (MW)

Vector. A quantity with magnitude and direction in the plane or in space, is defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

W

Whole numbers. The numbers 0, 1, 2, 3, ... See Illustration 1 in the Glossary.

Tables and Illustrations of Key Mathematical Properties, Rules, and Number Sets

Table 1. Common addition and subtraction situations³⁴

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown ³⁵
Put Together/ Take Apart³⁶	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare³⁷	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

³⁴ Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32–33).

³⁵ These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes or results in* but always does mean *is the same number as*.

³⁶ Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

³⁷ For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations³⁸

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$ and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are three bags with six plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need three lengths of string, each six inches long. How much string will you need altogether?	If 18 plums are shared equally into three bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into three equal pieces. How long will each piece of string be?	If eighteen plums are to be packed six to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are six inches long. How many pieces of string will you have?
Arrays,³⁹ Area⁴⁰	There are three rows of apples with six apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into three equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of six apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs three times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be three times as long?	A red hat costs \$18 and that is three times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

³⁸ The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

³⁹ The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in three rows and six columns. How many apples are in there? Both forms are valuable.

⁴⁰ Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The Properties of Operations

Here a , b , and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

<i>Associative property of addition</i>	$(a + b) + c = a + (b + c)$
<i>Commutative property of addition</i>	$a + b = b + a$
<i>Additive identity property of 0</i>	$a + 0 = 0 + a = a$
<i>Existence of additive inverses</i>	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
<i>Associative property of multiplication</i>	$(a \times b) \times c = a \times (b \times c)$
<i>Commutative property of multiplication</i>	$a \times b = b \times a$
<i>Multiplicative identity property of 1</i>	$a \times 1 = 1 \times a = a$
<i>Existence of multiplicative inverses</i>	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$.
<i>Distributive property of multiplication over addition</i>	$a \times (b + c) = a \times b + a \times c$

Table 4. The Properties of Equality

Here a , b , and c stand for arbitrary numbers in the rational, real, or complex number systems.

<i>Reflexive property of equality</i>	$a = a$
<i>Symmetric property of equality</i>	If $a = b$, then $b = a$.
<i>Transitive property of equality</i>	If $a = b$ and $b = c$, then $a = c$.
<i>Addition property of equality</i>	If $a = b$, then $a + c = b + c$.
<i>Subtraction property of equality</i>	If $a = b$, then $a - c = b - c$.
<i>Multiplication property of equality</i>	If $a = b$, then $a \times c = b \times c$.
<i>Division property of equality</i>	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
<i>Substitution property of equality</i>	If $a = b$, then b may be substituted for a in any expression containing a .

Table 5. Algorithms and the Standard Algorithms: Addition Example

Algorithm	Standard Algorithm (for efficiency)
$\begin{array}{r} 356 \\ +167 \\ \hline 400 \text{ (Sum of hundreds)} \\ 110 \text{ (Sum of tens)} \\ \underline{13} \text{ (Sum of ones)} \\ 523 \end{array}$	$\begin{array}{r} 11 \\ 356 \\ +167 \\ \hline 523 \end{array}$
<p>Note: All algorithms have a finite set of steps, are based on place value and properties of operations, and use single-digit computations.</p>	

Table 6. The Properties of Inequality

Here a , b , and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Illustration 1. The Number System

The Number System is comprised of number sets beginning with the Counting Numbers and culminating in the more complete Complex Numbers. The name of each set is written on the boundary of the set, indicating that each increasing oval encompasses the sets contained within. Note that the Real Number Set is comprised of two parts: Rational Numbers and Irrational Numbers.

