



**Alternate Academic Achievement
Standards (AAAS)
to the 2017 Massachusetts
Curriculum Frameworks
for Students with Disabilities**

**MATHEMATICS
Pre-Kindergarten–Grade 12**

Fall 2023



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Massachusetts Department of Elementary and Secondary Education

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Commissioner

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Introduction and Purpose

The Fall 2023 edition of the *Alternate Academic Achievement Standards to the Massachusetts Curriculum Frameworks in Mathematics* ("the Resource Guide") incorporates the curriculum content standards in the *2017 Mathematics Curriculum Framework*. The Resource Guides align achievement of grade-level standards with the requirements of the state's alternate assessment based on alternate academic achievement standards. The Resource Guide is intended to be used for students participating in the alternate assessment.

The Resource Guides are assessment guides for teachers who work with students with the most significant cognitive disabilities who are eligible to participate in the MCAS Alternate Assessment (MCAS-Alt).

The Resource Guide identifies standard-based outcomes called **entry points** for each standard to assist educators in teaching and assessing appropriately challenging standards-based academic skills and content that are aligned with grade-level standards, as required by law. Entry points also provide a roadmap for students to make steady progress toward standards at grade-level complexity.

In cases where students are unable to address entry points even at the lowest levels of complexity, due to the severity of their disability, teachers will use access skills that address early developmental communication and motor skills practiced during age-appropriate, standard-based math activities. Entry points and access skills are listed for each standard in this Resource Guide.

Resource Guides in English Language Arts (ELA), Mathematics, and Science and Technology/Engineering (STE) are available online at www.doe.mass.edu/mcas/alt/resources.html.

How Alternate Academic Achievement Standards Were Developed

The Department convened panels of experts in each content area, including content specialists, assessment experts, special educators familiar with students with the most significant cognitive disabilities, higher education faculty, parents and advocates, and members of the state's contractor team (see Acknowledgements on previous page). The panel reviewed the standards, unpacked the information, and identified the essence of the standard. Once panelists agreed upon the essence of the standard, entry points were created based on the standard and placed on a continuum from the least to the most complex. Teachers choose an entry point that assesses a challenging and attainable skill appropriate for each student.

How to Use the Alternate Academic Achievement Standards Guide (AAAS)

Figure 1 illustrates how to select entry points and access skills for the MCAS-Alt. Additional information on how to document student performance and progress throughout the school year can be found in the *Educator's Manual for MCAS-Alt*.

Organization of the Pre-Kindergarten through High School Mathematics Resource Guide

The *2017 Curriculum Framework in Mathematics* for students in **PreK through grade 8** is organized into **Domains** for the successive grades in which the domain is taught; for students in **high school**, Mathematics is organized into **Conceptual Categories**.

The **mathematics standards** in each grade are grouped into **clusters** and **cluster headings**. Domains and conceptual categories contain sets of **anchor standards**, followed by **grade-specific standards** that are coded by grade level and domain.

On the pages following each list of cluster headings and standards, the **entry points** are listed that describe outcomes based on alternate academic achievement standards linked with grade-specific standards.

Access skills are listed at the lowest grade level of each strand to address early developmental milestones within the context of activities aligned with grade-specific math content.

The table on page 7 indicates which domains and conceptual categories are required for alternate assessment in each grade. The table also shows related domains at lower grade levels that may be assessed for all grades.

Figure 1
How to Select Entry Points and Access Skills for the MCAS-Alt

Steps 1, 2 and 3

- Determine the grade in which the student is enrolled.
- Determine the domains/conceptual categories to be assessed.
- Conduct the MCAS-Alt Skills Survey for each domain/conceptual category to be assessed.

Steps 4 and 5

- Select two standards in grades 3–8: three standards in grade 10.
- Based on the results of the Skills Survey, select entry points in the domain/conceptual categories that the student can perform only occasionally (even with support).

Step 6

Once entry points are selected, design challenging instruction for the student.

Step 7, if needed

In cases where the MCAS-Alt Skills Survey indicates that the student cannot complete any of the skills even at the lowest level of complexity, the student should address access skills during standards-based activities in the domain.

Definition of Terms Used in the Alternate Academic Achievement Standards (AAAS)

The following terms are used to describe the information listed in the *Alternate Academic Achievement Standards (AAAS) to the 2017 Frameworks for Students with Disabilities—Mathematics*:

- ◆ **Access Skills** are developmental (communication or motor) skills addressed during standard-based activities in the content area being assessed. Access skills are listed at the lowest grade level in each mathematics domain or conceptual category in the Mathematics Resource Guide.
- ◆ **Clusters** are groups of related standards (e.g., “Define, evaluate, and compare functions” in the Functions domain)
- ◆ **Cluster headings** describe the overall knowledge and skills addressed by the standards in the cluster (i.e., the “essence”)
- ◆ **Conceptual Categories** are large groups of standards in a related area of Mathematics (e.g., Statistics and Probability) for students in high school.
- ◆ **Domains** are large groups of standards in a related area of Mathematics (e.g., Geometry) for students in PreK through grade 8.
- ◆ **Entry Points** are based on alternate academic achievement standards that are aligned with each standard or cluster of standards. They are intended for use by educators to instruct students eligible for participating in the alternate assessment.
- ◆ **Standards** define what students should understand and be able to do in each grade. Each standard is listed precisely as it appears in the *2017 Massachusetts Curriculum Framework for Mathematics*. Each standard is identified by an alpha-numeric code (e.g., Standard K.OA.A.5 - “Fluently add and subtract within 5”).

See a more detailed list of terms at <https://profile.measuredprogress.org/MCAS-Alt/login.aspx>.

2023 MCAS-Alt Mathematics

Progression from Pre-K through Grade 8

Domains	Grade Level									
	PK	K	1	2	3	4	5	6	7	8
<i>Counting and Cardinality</i>										
<i>Operations and Algebraic Thinking</i>					★	★				
<i>Number and Operations in Base Ten</i>							★			
<i>Number and Operations—Fractions</i>						★	★			
<i>The Number System</i>								★		
<i>Ratios and Proportional Relationships</i>									★	
<i>Expressions and Equations</i>										★
<i>Measurement and Data</i>					★					
<i>Geometry</i>									★	★
<i>Statistics and Probability</i>								★		

Progression for High School Conceptual Categories			High School (choice of three)
Domains			Conceptual Categories
Number and Operations Base Ten	The Number System		Number and Quantity
Operations and Algebraic Thinking	Expressions and Equations		Algebra
Number and Operations Fractions	Ratios and Proportional Relationships	Functions	Functions
Geometry			Geometry
Measurement and Data	Statistics and Probability		Statistics and Probability

★ = assessed by MCAS-Alt

***Counting and Cardinality**

*NOT ASSESSED

	Standards	Entry Points	Access Skills
Pre-K	Page 9	Pages 10 – 13	Pages 10 – 14
K	Page 15	Page 16	

CONTENT AREA Mathematics
DOMAIN Counting and Cardinality

Pre-Kindergarten

Cluster	Standards as written	
Know number names and the counting sequence.	PK.CC.A.1	Listen to and say the names of numbers in meaningful contexts.
	PK.CC.A.2	Recognize and name written numerals 0–10.
Count to tell the number of objects.	PK.CC.B.3	Understand the relationships between numerals and quantities up to ten.
Compare numbers.	PK.CC.C.4	Count many kinds of concrete objects and actions up to ten, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration.
	PK.CC.C.5	Use comparative language, such as more/less than, equal to, to compare and describe collections of objects.

ENTRY POINTS and ACCESS SKILLS for Counting and Cardinality Standards in Pre-K

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Know number names and the counting sequence.	<ul style="list-style-type: none"> ◆ Respond to materials as they are counted ◆ Shift focus from materials to speaker counting materials ◆ Grasp materials as they are counted ◆ Release materials as they are counted ◆ Give materials as they are counted ◆ Move objects as they are counted ◆ Orient objects as they are counted (e.g., turn flowerpots upright) ◆ Manipulate objects with two hands as they are counted ◆ Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted ◆ Use one object to act on another as objects are counted (e.g., use a pointer to tap) 	<ul style="list-style-type: none"> ◆ Identify numerals ◆ Identify number words ◆ Count by ones to 3 ◆ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 3 ◆ Answer yes/no questions about counting 	<ul style="list-style-type: none"> ◆ Count by ones to 5, without objects ◆ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 5 ◆ Sing songs/recite rhymes that involve counting up to 5. 	<ul style="list-style-type: none"> ◆ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 10 ◆ Count by ones up to 10 without objects ◆ Sing songs/recite rhymes that involve counting up to 10.

ACCESS SKILLS (continued) for Counting and Cardinality Standards in Pre-K

Less Complex

More Complex

<u>ACCESS SKILLS</u> <u>The student will:</u>		<u>ENTRY POINTS</u> <u>The student will:</u>
Know number names and the counting sequence. (continued)	<ul style="list-style-type: none"> ♦ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall) ♦ Construct using materials that have been counted in sequence (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence activity ♦ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Sustain counting sequence activity through response ♦ Gain attention in counting sequence activity ♦ Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Attend visually, aurally, or tactilely to objects as they are counted 	

ENTRY POINTS and ACCESS SKILLS for Counting and Cardinality Standards in Pre-K

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Count to tell the number of objects.	<ul style="list-style-type: none"> ◆ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted) ◆ Follow directions in counting sequence activities (e.g., follow direction to “put the pencils in the box” as the teacher counts) ◆ Sustain activity through response during a counting activity 	<ul style="list-style-type: none"> ◆ Place a given number of manipulatives/ objects up to 3 ◆ Use fingers to represent a number up to 3 ◆ Match number names or numerals to small quantities of objects up to 3 ◆ Choose from two numerals the numeral that represents the number of manipulatives 	<ul style="list-style-type: none"> ◆ Place given number of manipulatives/ objects up to 5 ◆ Use fingers to represent a number up to 5 ◆ Match number names or numerals to small quantities of objects up to 5 	<ul style="list-style-type: none"> ◆ Place a given number of manipulatives/ objects up to 10 ◆ Use fingers to represent a number up to 10 ◆ Match number names or numerals to small quantities of objects up to 10

ENTRY POINTS and ACCESS SKILLS for Counting and Cardinality Standards in Pre-K

Less Complex

More Complex

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Compare numbers.	<ul style="list-style-type: none"> ◆ Respond to materials being compared by quantity or size ◆ Track materials being compared by quantity or size ◆ Shift focus from materials being compared by quantity or size to speaker ◆ Grasp materials being compared by quantity or size ◆ Release materials being compared by quantity or size ◆ Give materials being compared by quantity or size ◆ Move materials being compared by quantity or size ◆ Orient materials being compared by quantity or size ◆ Manipulate materials being compared by quantity or size ◆ Locate objects partially hidden or out of sight to compare by quantity or size to another set 	<ul style="list-style-type: none"> ◆ Demonstrate 1:1 correspondence by counting up to 3 objects, starting from one ◆ Distinguish between same/different quantities of similar objects ◆ Organize objects in a group to represent a quantity up to 3 ◆ Match quantity to quantity name ◆ Select (by counting) which group of objects has 1, 2, or 3 items 	<ul style="list-style-type: none"> ◆ Demonstrate 1:1 correspondence by counting up to 5 objects, starting from one ◆ Select by counting which group of objects has 1–5 items ◆ Identify the correct amount of scattered objects without counting, up to 3 	<ul style="list-style-type: none"> ◆ Demonstrate 1:1 correspondence by counting up to 7 objects, starting from one ◆ Select by counting which group of objects has 1–7 items. ◆ Identify the correct amount of scattered objects without counting, up to 5

ACCESS SKILLS (continued) for Counting and Cardinality Standards in Pre-K

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Compare numbers. (continued)	<ul style="list-style-type: none"> ◆ Construct two objects using materials from two sets of materials being compared by quantity or size (e.g., build two block towers, with one set of 3 blocks and one with 5 blocks) ◆ Turn on voice-generating device to participate in an activity to compare materials by quantity or size ◆ Imitate action with materials being used to compare quantity or size ◆ Sustain activity comparing objects by size or quantity through response ◆ Gain attention in activity comparing objects by size and quantity (e.g., raise hand during comparison lesson on white board) ◆ Make a request in an activity comparing materials by size or quantity ◆ Choose from (or beyond) an array of two in an activity to compare materials by quantity ◆ Attend visually, aurally, or tactilely to materials being compared by quantity or size 	

CONTENT AREA Mathematics
DOMAIN Counting and Cardinality

Kindergarten

Cluster	Standards as written	
Know number names and the count sequence.	K.CC.A.1	Count to 100 by ones and by tens.
	K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
	K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).
Count to tell the number of objects.	K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality.
	K.CC.B.4a	When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
	K.CC.B.4b	Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
	K.CC.B.4c	Understand that each successive number name refers to a quantity that is one larger.
	K.CC.B.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Compare numbers.	K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
	K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.

ENTRY POINTS for Counting and Cardinality Standards in Kindergarten

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Know number names and the count sequence.	<ul style="list-style-type: none"> Count to 20 by ones Starting with any number greater than one, count forward by ones to 5 Write numbers from 1-5 Match spoken number names and written numerals (e.g., one/1, 2/two), up to 15 Demonstrate 1:1 correspondence between objects by counting up to 15 objects, starting from one <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Count to 50 by ones Starting with any number greater than one, count forward by ones to 10 Write numbers from 1-10 Match spoken number names and written numerals (e.g., one/1, 2/two), up to 20 Demonstrate 1:1 correspondence between objects by counting up to 20 objects, starting from one 	<ul style="list-style-type: none"> Count to 50 by tens Starting with any number greater than one, count forward by ones to 20 Write numbers from 1-15 Demonstrate 1:1 correspondence between objects by counting up to 50 objects, starting from one <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Count to tell the number of objects.	<ul style="list-style-type: none"> Count to answer "how many?" (line, array, circle) using various arrangements of up to 3 objects Select objects from a larger group, up to 3 	<ul style="list-style-type: none"> Count to answer "how many?" (line, array, circle) using various arrangements of up to 5 objects 	<ul style="list-style-type: none"> Count to answer "how many?" using various arrangements (line, array, circle) of up to 10 objects
Compare numbers.			<ul style="list-style-type: none"> Compare two written numerals between 1 and 5

Operations and Algebraic Thinking

	Standards	Entry Points	Access Skills
Pre-K	Page 18	Page 19	Pages 19 – 20
K	Page 21	Page 22	
1	Page 23	Pages 24 – 26	
2	Page 27	Pages 28 – 30	
3	Pages 31 – 32	Pages 33 – 35	
4	Page 36	Pages 37 – 38	
5	Page 39	Page 40	

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Pre-Kindergarten

Cluster	Standards as written	
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	PK.OA.A.1	Use concrete objects to model real-world addition (putting together) and subtraction (taking away) problems up through five.

ENTRY POINTS and ACCESS SKILLS for Operations and Algebraic Thinking Standards in Pre-K

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	<ul style="list-style-type: none"> ◆ Respond to materials to be added, subtracted, and/or counted ◆ Track object as it is added, subtracted, and/or counted from set ◆ Shift focus on materials as they are added, subtracted, and/or counted ◆ Grasp materials to be added, subtracted, and/or counted ◆ Release materials to be added, subtracted, and/or counted ◆ Give materials to be added, subtracted, and/or counted ◆ Move materials to be added, subtracted, and/or counted ◆ Orient objects/ materials added, subtracted, and/or counted (e.g., turn flowerpots upright as they are added, subtracted, and/or counted) 	<ul style="list-style-type: none"> ◆ Combine and take apart sets of objects to represent the concept of addition and subtraction ◆ Add ("put together") and subtract ("take away") using up to 2 manipulatives or objects ◆ Answer yes/no questions about "adding to" or "taking from" ◆ Add one or two more to a set of objects (e.g., using manipulatives) 	<ul style="list-style-type: none"> ◆ Create sets of "more," starting with 2 or 3 objects ◆ Create sets of "less" starting with 3 objects ◆ Compare groupings of various objects, using comparative terms up to 4 ◆ Show "one more" starting with a given number of objects ◆ Show "one less" starting with a given number of objects ◆ Illustrate concept of more or less ◆ Identify sets of more or less/fewer 	<ul style="list-style-type: none"> ◆ Demonstrate "add to" using 5 or more objects or manipulatives ◆ Demonstrate "subtract from" using 5 or more manipulatives or objects

ACCESS SKILLS (continued) for Operations and Algebraic Thinking Standards in Pre-K

Less Complex

More Complex

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Understand addition as putting together and adding to, and subtraction as taking apart and taking from (continued)	<ul style="list-style-type: none"> ◆ Locate objects partially hidden or out of sight, to add to a set ◆ Use one object to act on another in an adding, subtracting, and/or counting activity (e.g., use a net to add or subtract blocks to sets) ◆ Construct/deconstruct or assemble/disassemble object by adding or subtracting pieces (e.g., add or subtract blocks from tower) ◆ Turn device on/off during an addition, subtraction, and/or counting activity (e.g., turn on voice-generating device to indicate add) ◆ Imitate action to add, subtract, and/or count ◆ Initiate cause-and-effect response (e.g., use switch to activate an adding, subtracting, and/or counting cause-and-effect program) ◆ Sustain addition, subtraction, and/or counting activity through response (e.g., when reading about adding, subtracting, and/or counting stops, vocalizes to request more) ◆ Gain attention in an addition, subtraction, and/or counting activity ◆ Make a request during adding, subtracting, and/or counting activity (e.g., to ask for help or request a turn to add or subtract materials) ◆ Choose from an array of two in an addition, subtraction, and/or counting lesson ◆ Attend visually, aurally, or tactilely to materials to be added, subtracted, and/or counted 	

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Kindergarten

Cluster	Standards as written	
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	K.OA.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, or equations.
	K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
	K.OA.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
	K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
	K.OA.A.5	Fluently add and subtract within 5.

ENTRY POINTS for Operations and Algebraic Thinking Standards in Kindergarten

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	<ul style="list-style-type: none"> ◆ Add ("put together") 1, 2, and 3 fingers, manipulatives, or objects ◆ Subtract ("take away") 1, 2, and 3 fingers, manipulatives, or objects ◆ "Add one" or "take away one" using objects or pictures from sets of up to 3 ◆ Represent addition and subtraction word problems with up to 3 objects or manipulatives ◆ Decompose a number in more than one way using objects or manipulatives (e.g., $3 = 1 + 2$; $3 = 2 + 1$) ◆ Find the number that makes 3 when using the numbers 1 or 2 added to the given number (using fingers or sounds, e.g., claps) ◆ Find the number that makes 5 when added to a given number using fingers or manipulatives ◆ Count up to 5 objects to answer questions about how many altogether, using manipulatives ◆ Add ("put together") and subtract ("take away") within 3 using objects or manipulatives <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add ("put together") and subtract ("take away") using the numbers 1–5 (using sounds, e.g., claps) ◆ Represent addition and subtraction using the numbers 1–5 (e.g., using manipulatives, acting out situations) ◆ "Add two" or "take away two" from sets of up to 5 using objects or pictures ◆ Represent addition and subtraction word problems with up to 5 objects/manipulatives, or with drawings ◆ Decompose numbers up to 5 in more than one way using drawings or objects (e.g., $5 = 2 + 3$, $5 = 4 + 1$) ◆ Find the number that makes 5 when added to a given number 1–4, (using manipulatives, objects, or drawings) ◆ Find the number that makes 5 when added to the given number and record the answer using a drawing or manipulatives ◆ Add ("put together") and subtract ("take away") within 5 using objects and manipulatives ◆ Count up to 10 objects to answer questions about how many altogether using manipulatives 	<ul style="list-style-type: none"> ◆ Select the symbol (+) to represent addition and the symbol (-) to represent subtraction upon request ◆ "Add two" or "take away two" from sets of up to 10 using objects or pictures ◆ Solve addition and subtraction problems by manipulating objects ◆ Express the meanings of "add" (make more) and "subtract" (make less) ◆ Represent addition and subtraction word problems with up to 10 objects/manipulatives, or with drawings ◆ Match numerals with up to 10 objects ◆ Decompose numbers less than or equal to 10 in more than one way and express as an equation (e.g., $7 = 2 + 5$ and $7 = 4 + 3$) ◆ Demonstrate two ways to decompose a set of objects to create the inverse (e.g., $1 + 2 = 3$ to $3 - 1 = 2$) ◆ Find the number that makes 10 when added to a given number (e.g., using objects) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 1

Cluster	Standards as written	
Represent and solve problems involving addition and subtraction.	1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations (number sentences) with a symbol for the unknown number to represent the problem.
	1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Understand and apply properties of operations and the relationship between addition and subtraction.	1.OA.B.3	Apply properties of operations to add. <i>For example, when adding numbers order does not matter. If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known (Commutative property of addition). To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$ (Associative property of addition). When adding zero to a number, the result is the same number (Identity property of zero for addition).</i>
	1.OA.B.4	Understand subtraction as an unknown-addend problem. <i>For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.</i>
Add and subtract within 20.	1.OA.C.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
	1.OA.C.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use mental strategies such as counting on; making 10 (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a 10 (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).
Work with addition and subtraction equations.	1.OA.D.7	Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. <i>For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.</i>
	1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = \square - 3$, $6 + 6 = \square$.</i>

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 1

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Represent and solve problems involving addition and subtraction.	<ul style="list-style-type: none"> ◆ Add and subtract up to three one-digit numbers using manipulatives or objects ◆ Identify sets containing more, less/fewer ◆ Show "add to", using manipulatives and examples up to 7 ◆ Show "take from", using manipulatives and examples up to 7 ◆ Show "put together," with manipulatives and examples up to 7 ◆ Show "take apart," with manipulatives and examples up to 7 ◆ Show "compare" with manipulatives and examples up to 7 ◆ Express "putting together" and "taking apart" with the appropriate language to describe any aspect of addition (plus, combined, added, more) ◆ Express "taking apart" with the appropriate language to describe any aspect of subtraction (minus, take away, less, remove, difference) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Solve problems involving addition or subtraction of one-digit numbers represented by groups of objects, with a result no greater than 10, using pictures or manipulatives 	<ul style="list-style-type: none"> ◆ Add and subtract up to three one-digit numbers that total no more than 10 ◆ Use conventional mathematical signs and symbols to represent mathematical relationships in a word problem; i.e., (+), (-), and (=) ◆ Create and/or solve an equation, based on a word problem using addition and/or subtraction within 20 (e.g., Marie needs 16 chairs for her party, she finds 9 in the garage. How many chairs does she need? $? + 9 = 16$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 1

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand and apply properties of operations and the relationship between addition and subtraction	<ul style="list-style-type: none"> ◆ Combine objects to demonstrate addition ("plus," "combined with," "more," "join") ◆ Take groups of objects apart to demonstrate subtraction ("minus," "take away," "less," "remove," "difference") ◆ Group objects to demonstrate the commutative property of addition (e.g., one object plus two objects is the same as two objects plus one object) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Show the commutative property of addition use pairs of numbers 1–5 (e.g., if $1 + 4 = 5$ is known, then $4 + 1 = 5$ is also known) ◆ Group objects to demonstrate the associative property of addition 	<ul style="list-style-type: none"> ◆ Show the associative property of addition use up to three one-digit numbers 1–10 where the first two digits are added together (e.g., $5 + 3 + 2 = 5 + 5 = 10$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Add and subtract within 20.	<ul style="list-style-type: none"> ◆ Create equal sets using pictures or objects ◆ Match numerals with up to 5 objects ◆ Indicate the number that results when adding one more to a given number, use manipulatives or visual representations or a number line (e.g., to find $1 + 4$, count 4 objects, then one more) ◆ Subtract 1 from numbers up to 5 using concept of "one less" <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add objects by grouping into combinations up to 5 (using manipulatives, fingers, etc.) ◆ Add and subtract numbers within 5 by using the relationship between addition and subtraction (e.g., if $2 + 3 = 5$, then we know that $5 - 2 = 3$) ◆ Match numeral with up to 10 objects 	<ul style="list-style-type: none"> ◆ Add and subtract one-digit numbers in number sentence ◆ Solve problems involving addition and subtraction using the numbers 1–10 ◆ Add and subtract within 10 using strategies such as counting on and using the relationship between addition and subtraction (e.g., knowing that $8 + 2 = 10$, we know that $10 - 8 = 2$) ◆ Match numeral with up to 15 objects <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Operations and Algebraic Thinking in Grade 1

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Work with addition and subtraction equations.	<ul style="list-style-type: none"> ◆ Identify the terms and corresponding symbols for addition (+) and subtraction (-) ◆ Show the concept of trading equal amounts ◆ Add one object to each set express result ◆ Match one-to-one equivalents (e.g., put one object in each of multiple sets, match pairs of objects) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Solve simple addition and subtraction number sentences, using pictures, objects, or manipulatives ◆ Make comparisons between sets ◆ Determine missing objects or quantities within sets ◆ Create sets to demonstrate the meaning of "equal to" ◆ Create sets that have the same number of objects 	<ul style="list-style-type: none"> ◆ Solve simple addition and subtraction number sentences, using symbols or numbers (e.g., $2 + ? = 4$) ◆ Show equalities in number sentences (e.g., $3 + 1 = 2 + 2$) ◆ Represent the relationships between groupings of objects using symbol of equality ◆ Express appropriate terms and corresponding symbol for "equal to" ◆ Represent mathematical relationships using conventional mathematical signs and symbols ◆ Solve number sentences that express relationships involving addition and subtraction within 10 <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 2

Cluster	Standards as written	
Represent and solve problems involving addition and subtraction.	2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
Add and subtract within 20.	2.OA.B.2	Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two single-digit numbers and related differences. <i>For example, the sum $6 + 5 = 11$ has related differences of $11 - 5 = 6$ and $11 - 6 = 5$.</i>
Work with equal groups of objects to gain foundations for multiplication.	2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
	2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to five rows and up to five columns; write an equation to express the total as a sum of equal addends.

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 2

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Represent and solve problems involving addition and subtraction.	<ul style="list-style-type: none"> Identify the terms and corresponding symbols for addition (+) and subtraction (-) Add and subtract numbers within 20 using manipulatives Represent addition and subtraction word problems within 20 using manipulatives Manipulate number sentences in a given problem to show that the same sum can be achieved by adding numbers in different arrangements up to 20 (e.g., $1 + 2 = 3$ and $2 + 1 = 3$) Represent simple addition and subtraction number sentences with manipulatives Create number sentences up to 20 by manipulating objects Solve word one-step problems using addition and subtraction within 20 to "add to" involving unknowns in all positions with manipulatives Solve word problems using addition and subtraction within 20 to "put together/take apart" involving unknowns in all positions with manipulatives Solve one-step word problems using addition and subtraction within 20 to "take from" involving unknowns in all positions with manipulatives 	<ul style="list-style-type: none"> Identify terms and/or symbols for +, -, = Add and subtract numbers within 50 using manipulatives and drawings Represent one-step addition and subtraction word problems within 50 using manipulatives and drawings Express the inverse relationship between addition and subtraction within 50 using number sentences Identify common phrases used to indicate the need for addition and subtraction Create number sentences with unknown numbers up to 50 using pictures/objects Solve word one-step problems using addition and subtraction within 50 to "add to" involving unknowns in all positions with manipulatives and drawings Solve word problems using addition and subtraction within 50 to "put together/take apart" involving unknowns in all positions with manipulatives and drawings Solve one-step word problems using addition and subtraction within 50 to "take from" involving unknowns in all positions with manipulatives and drawings 	<ul style="list-style-type: none"> Create and/or solve an equation, based on a word problem using addition and/or subtraction within 100 (e.g., Juan is holding a fundraiser for the playground. He needs 20 tables for lunch. The local school gave him 9. How many more does he need. ? $+ 9 = 20$ or $20 - 9 = ?$) Solve double digit addition problems Add and subtract two-digit numbers within 100 Add and subtract numbers within 100 using manipulatives or drawings <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 2

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Represent and solve problems involving addition and subtraction. (continued)	<ul style="list-style-type: none"> Solve one-step word problems using addition and subtraction within 20 to “compare” involving unknowns in all positions with manipulatives <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Solve one-step word problems using addition and subtraction within 50 to “compare” involving unknowns in all positions with manipulatives Create and solve number sentences up to 20 using manipulatives and/or drawings 	
Add and subtract within 20.	<ul style="list-style-type: none"> Demonstrate addition within 5 as “putting together” and subtraction as “taking away” using manipulatives and drawings <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Demonstrate addition within 10 as “putting together” and subtraction as “taking away” using manipulatives and drawings 	<ul style="list-style-type: none"> Demonstrate addition within 20 as “putting together” and subtraction as “taking away” using manipulatives and drawings <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Work with equal groups of objects to gain foundations for multiplication.	<ul style="list-style-type: none"> Create two equal groups from an even number of objects Create a 2xN array by arranging an even number of objects into groups of twos (N can be 1-10) Skip count objects by rows arranged in a 2xN array to demonstrate repeated addition (e.g., A carton of eggs with 2 rows of 6 eggs = $2 + 2 + 2$) Group objects by multiples of two <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>		<ul style="list-style-type: none"> Create sets to demonstrate the meaning of “equal to” Group and count by 2’s up to 20 Group and count by 5’s up to 20 Group and count by 10’s up to 20 Determine if a quantity is odd or even by pairing two groups of up to 20 objects, using manipulatives and drawings Match two equal groupings with odd or even quantities with appropriate numeral to represent actual quantities up to 10 Use manipulatives or models to add the number of objects in an array with up to 5 rows and 5 columns

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 2

Less Complex

More Complex

The student will:

The student will:

The student will:

Work with equal groups of objects to gain foundations for multiplication.
(continued)

- ♦ Equally distribute an even number of objects up to 20 into 2–4 groups
- ♦ Group and count by 2s and 5s

Continue to address skills and concepts that approach grade-level expectations in this cluster

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 3

Cluster	Standards as written	
Represent and solve problems involving multiplication and division.	3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in five groups of seven objects each. <i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i>
	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.</i>
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.</i>
Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B.5	Apply properties of operations to multiply. <i>For example: When multiplying numbers order does not matter. If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known (Commutative property of multiplication); The product $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$ then $15 \times 2 = 30$, or by $5 \times 2 = 10$ then $3 \times 10 = 30$ (Associative property of multiplication); When multiplying two numbers either number can be decomposed and multiplied; one can find 8×7 by knowing that $7 = 5 + 2$ and that $8 \times 5 = 40$ and $8 \times 2 = 16$, resulting in $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (Distributive property); When a number is multiplied by 1 the result is the same number (Identity property of 1 for multiplication).</i>
	3.OA.B.6	Understand division as an unknown-factor problem. <i>For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.</i>
Multiply and divide within 100.	3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of grade 3, know from memory all products of two single-digit numbers and related division facts. <i>For example, the product $4 \times 7 = 28$ has related division facts $28 \div 7 = 4$ and $28 \div 4 = 7$.</i>

Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D.8	Solve two-step word problems using the four operations for problems posed with whole numbers and having whole number answers. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding. ¹
	3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>

¹ Students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 3

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Represent and solve problems involving multiplication and division.	<ul style="list-style-type: none"> ◆ Skip count the number of objects in an array to demonstrate repeated addition ◆ Represent and solve problems involving repeated addition ◆ Group objects together by 2s using manipulatives or drawings <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Represent repeated addition within 20 using two equal groups of objects (e.g., 2 groups of 3 is the same as $3 + 3$) ◆ Group up to 10 objects in an array that demonstrates the concept of repeated addition ◆ Identify the number sentence that demonstrates the concept of multiplication using a given array of objects ◆ Solve simple multiplication number sentences with unknown numbers, using pictures, objects, and/or manipulatives 	<ul style="list-style-type: none"> ◆ Represent repeated addition within 30 using two equal groups of objects (e.g., 2 groups of 6 is the same as $6 + 6$) ◆ Group up to 20 objects in an array that demonstrates the concept of repeated addition ◆ Create a number sentence showing multiplication using arrays of objects and manipulatives ◆ Identify common phrases used to indicate the need for multiplication such as "product", multiplication and "groups" ◆ Solve multiplication problems involving unknown variables in simple one-digit number sentences <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Understand properties of multiplication and the relationship between multiplication and division.	<ul style="list-style-type: none"> ◆ Create groups of objects to demonstrate the commutative property of multiplication (e.g., 4 groups of 2 squares equals 2 groups of 4 squares) ◆ Illustrate the concept of multiplication using groups of objects ◆ Share up to 10 objects equally between 2 people (divide objects into 2 equal groups) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Match written problems that demonstrate the commutative properties of multiplication (e.g., $2 \times 3 = 6$ and $3 \times 2 = 6$) to visual representation ◆ Demonstrate the relationship between multiplication and division using manipulatives (e.g., $8 \div 2$ is 8 objects divided into 2 groups of 4, which is equal to 4 groups of 2) ◆ Determine the unknown quantity in a multiplication equation (within 10) using manipulatives and drawings (e.g., how many groups of 4 objects is equal to 4?) 	<ul style="list-style-type: none"> ◆ Demonstrate the commutative property of multiplication and addition using arrays (e.g., 2 groups of 3 objects is equal to 3 groups of 2 objects) ◆ Create number sentences to demonstrate the relationship between multiplication and division (e.g., $8 \div 2 = 4$ is the same as $4 \times 2 = 8$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

MATHEMATICS

Operations and Algebraic Thinking Standards in Grade 3

Less Complex

More Complex

<p>Understand properties of multiplication and the relationship between multiplication and division. (continued)</p>		<ul style="list-style-type: none"> ◆ Illustrate the concept of division using groups of objects ◆ Visually represent problem situations by sharing up to 20 objects equally between 2 and 5 people ◆ Match written problems that demonstrate the distributive property (e.g., 4 groups of 2 squares equals 2 groups of 4 squares) 	<ul style="list-style-type: none"> ◆ Determine the unknown quantity in a multiplication equation (within 20) using manipulatives and drawings (e.g., how many groups of objects is equal to 15?) ◆ Identify common phrases used to indicate the need for multiplication and division ◆ Write a number sentence representing up to 30 objects divided into equal groups of 2, 5, or 10 ◆ Demonstrate the commutative property of multiplication using manipulatives ◆ Illustrate division by making equal-sized groups using models (e.g., 2 equal-sized groups up to 10) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
<p>Multiply and divide within 100.</p>	<ul style="list-style-type: none"> ◆ Solve multiplication problems with a multiplier 1, 2, 5, or 10 ◆ Solve division problems within 100 with a divisor of 1, 2, 5, or 10 <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Solve multiplication problems with a multiplier 1, 2, 3, 4, 5, or 10 ◆ Solve division problems within 100 with a divisor of 1, 2, 3, 4, 5, or 10 	<ul style="list-style-type: none"> ◆ Solve multiplication problems with multipliers of up to 10 ◆ Solve division problems within 100 with divisors of up to 10 <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS to Standards for Operations and Algebraic Thinking in Grade 3

← Less Complex

More Complex →

<p>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p>	<ul style="list-style-type: none"> ♦ Solve one-step word problems using addition or subtraction ♦ Identify the required operation to solve one-step word problem ♦ Reproduce a given number pattern ♦ Reproduce a given shape pattern ♦ Extend simple repeating shape patterns, given a model or example ♦ Extend simple repeating number patterns, given a model or example ♦ Use rounding strategies to make estimates ♦ Predict what will come next in a shape pattern with three shapes (e.g., square, circle, triangle) ♦ Extend a simple addition or subtraction pattern (e.g., adding by 2s, subtracting by 3s) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ♦ Identify the missing component in a multiplication number sentence within 20, using symbols or objects (e.g., identify the array that matches the sentence $2 \times ? = 10$) ♦ Solve one-step word problems using multiplication or division ♦ Identify a number or shape pattern ♦ Create repeating shape patterns given a rule (e.g., given the rule ABCABC, student draws $\nabla \diamond \square \nabla \diamond \square$) 	<ul style="list-style-type: none"> ♦ Choose whether an estimate or exact amount is needed in a given situation ♦ Identify the missing component in a multiplication number sentence within 30, using symbols or objects (e.g., identify the array that matches the sentence $5 \times ? = 15$) ♦ Solve two-step word problems using any of the four operations ♦ Use estimation to determine the reasonableness of a solution to a one-step word problem ♦ Represent a word problem using a number sentence and appropriate symbols ($+$, $-$, \times, \div, $=$) ♦ Use estimation to approximate the solution to a one-step word problem ♦ Create a pattern based on a given rule ♦ Identify the rule of a given number or shape pattern ♦ Create a simple addition or subtraction pattern <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
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CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 4

Cluster	Standards as written	
Use the four operations with whole numbers to solve problems.	4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
	4.OA.A.3	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
	4.OA.A.3a	Know multiplication facts and related division facts through 12×12 .
Gain familiarity with factors and multiples.	4.OA.B.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
Generate and analyze patterns.	4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i>

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 4

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use the four operations with whole numbers to solve problems.	<ul style="list-style-type: none"> Identify the missing number in an addition or subtraction problem using manipulatives or drawings Solve a one-step word problem involving addition, subtraction, multiplication, or division Visually illustrate a one-step addition or subtraction problem with an unknown number using manipulatives or drawings Determine the reasonableness of an answer to an addition or subtraction problem using estimation strategies of "more" and "less" Replace unknown numbers with given values in two-digit by two-digit addition/subtraction sentences <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Identify the missing number in number sentences involving addition and subtraction (e.g., $25 - ? = 21$) Visually represent single-digit multiplication problems using strategies such as equal-sized groups, repeated addition, equal-sized jumps on a number line, or area models Solve number sentences that represent one-step multiplication and division word problems Solve number sentences derived from one-step addition or subtraction word problems using a letter or symbol for the unknown quantity Replace unknown numbers with given values in simple mathematical sentences (e.g., solve $2 \times ?$ If $? = 2, 3$, or 4) Determine the reasonableness of an answer to multiplication problems using estimation strategies of repeated addition 	<ul style="list-style-type: none"> Identify the missing number in number sentences involving multiplication and division (e.g., $10 \div ? = 5$) Solve a two-step word problem Solve number sentences derived from one-step word problems using letters or symbols for the unknown quantity Determine the reasonableness of an answer to division problems using estimation strategies (e.g., successive subtraction) Visually represent division problems with single-digit divisors using successive subtraction, equal-sized jumps on a number line, sharing, or partitioning <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Gain familiarity with factors and multiples.	<ul style="list-style-type: none"> Find multiples of 2 using manipulatives or drawings Find factors within 10 using manipulatives, tables, or drawings <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Find multiples of 2 and/or 3 using manipulatives or drawings Find factors within 20 using manipulatives, tables, or drawings 	<ul style="list-style-type: none"> Find multiples of 2, 3, and 5 using manipulatives, drawings, or symbols Find factors within 50 using manipulatives, tables, or drawings Identify prime numbers and numbers that can be factored <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 4

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Generate and analyze patterns.	<ul style="list-style-type: none"> ◆ Reproduce a given number pattern (e.g., given 2, 4, 2, 4, 2, 4 student recreates the pattern using manipulatives) ◆ Reproduce a given shape pattern ◆ Extend repeating shape patterns, given a model or example ◆ Extend simple repeating number patterns, given a model or example <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify a number or shape pattern ◆ Predict what will come next in a shape pattern with three or more shapes (e.g., square, circle, triangle) ◆ Extend a simple repeating number pattern (e.g., write the missing numbers given 1, 2, 3, 1, 2, 3, __, __) 	<ul style="list-style-type: none"> ◆ Create a pattern based on a given rule ◆ Identify the rule of a given number or shape pattern ◆ Create a pattern based on a given rule (e.g., starting with 2 with the rule "add 2, subtract 2" give the next four numbers) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 5

Cluster	Standards as written	
Write and interpret numerical expressions.	5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols, e.g., $(6 \times 30) + (6 \times \frac{1}{2})$.
	5.OA.A.2	Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. <i>For example, express the calculation "Add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i>
Analyze patterns and relationships.	5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. <i>For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</i>

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 5

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Write and interpret numerical expressions.	<ul style="list-style-type: none"> ◆ Solve numerical expressions in a given problem using the appropriate operation(s) ◆ List the Order of Operations given a numerical expression involving addition and subtraction (e.g., $10 - 5 + 2 = ?$) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Use the Order of Operations to solve numerical expressions involving addition and subtraction ◆ Use the Order of Operations to create a numerical expression based on a real-world problem involving addition before subtraction ◆ Use the Order of Operations to solve two-step numerical expressions 	<ul style="list-style-type: none"> ◆ Use the Order of Operations to evaluate numerical expressions involving addition, subtraction, and multiplication ◆ Use the Order of Operations to create a numerical expression based on a real-world problem involving multiplication before addition ◆ Use the Order of Operations to solve numerical expressions containing grouping symbols (e.g., $3 \times (2 - 1) = 3 \times 1 = 3$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Analyze patterns and relationships.	<ul style="list-style-type: none"> ◆ Extend a simple numerical pattern ◆ Count by twos to create a numerical pattern <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify a numerical pattern ◆ Extend a numerical pattern using a rule for addition and subtraction ◆ Count by twos and fives to create a numerical pattern ◆ Identify what is missing from a numerical pattern 	<ul style="list-style-type: none"> ◆ Explain the rule of a given numerical pattern ◆ Create a numerical pattern using a rule for addition, subtraction, or multiplication ◆ Count by twos, fives, tens, and hundreds to create a numerical pattern <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

Number and Operations in Base Ten

	Standards	Entry Points	Access Skills
K	Page 42	Page 43	Pages 43 – 44
1	Page 45	Pages 46 – 47	
2	Page 48	Page 49 – 50	
3	Page 51	Page 52	
4	Page 53	Page 54	
5	Page 55	Pages 56 – 57	

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Kindergarten		
Cluster	Standards as written	
Work with numbers 11–19 to gain foundations for place value.	K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

ENTRY POINTS and ACCESS SKILLS for Number and Operations in Base Ten Standards in Kindergarten

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u> The student will:	<u>ENTRY POINTS</u> The student will:	<u>ENTRY POINTS</u> The student will:
Work with numbers 11–19 to gain foundations for place value.	<ul style="list-style-type: none"> ◆ Respond to materials as they are counted ◆ Shift focus from materials to speaker counting materials ◆ Grasp materials as they are counted ◆ Release materials as they are counted ◆ Give materials as they are counted ◆ Move objects as they are counted ◆ Orient objects as they are counted (e.g., turn flowerpots upright) ◆ Manipulate objects with two hands as they are counted ◆ Locate objects partially hidden, or out of sight, to add or subtract to a collection of objects to be counted ◆ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ◆ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall) 	<ul style="list-style-type: none"> ◆ Count by ones up to 10 ◆ Represent a number of objects (up to 5) with a written numeral (with 0 representing a count of no objects). ◆ Compose numbers from 1 to 9 to create 10 and record each composition by using objects. ◆ Decompose 10 into two numbers between 1 and 9 and record each decomposition by using objects ◆ Answer yes/no questions related to numbers, quantities or counting 	<ul style="list-style-type: none"> ◆ Compose numbers from 1 to 9 to create 10, record each composition by using objects and/or drawings. ◆ Decompose 10 into two numbers between 1 and 9; record each decomposition by using objects and/or drawings ◆ Compose numbers from 1 to 9 to create 10 and record each composition by using objects, drawings and/or equations (e.g., $1 + 9 = 10$) ◆ Decompose 10 into two numbers between 1 and 9 and record each decomposition by using objects, drawings and/or equations (e.g., $10 - 1 = 9$)

ACCESS SKILLS (continued) for Number and Operations in Base Ten Standards in Kindergarten

← Less Complex

More Complex →

ACCESS SKILLS The student will:

ENTRY POINTS The student will:

Work with numbers 11–19 to gain foundations for place value.
(continued)

- ◆ Construct using materials that have been counted in sequence (e.g., tower of blocks)
- ◆ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names)
- ◆ Imitate action in counting sequence activity
- ◆ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program)
- ◆ Sustain counting sequence activity through response
- ◆ Gain attention in counting sequence activity
- ◆ Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game)
- ◆ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted)
- ◆ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted)
- ◆ Follow directions in counting sequence activities (e.g., follow direction to “Put the pencils in the box” as the teacher counts)
- ◆ Attend visually, aurally, or tactilely to objects as they are counted

CONTENT AREA Mathematics

DOMAIN Number and Operations in Base Ten

Grade 1

Cluster	Standards as written	
Extend the counting sequence.	1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Understand place value.	1.NBT.B.2	Understand that the two digits of a two-digit number represent amounts of tens and ones.
	Understand the following (2a, 2b, and 2c) as special cases:	
	1.NBT.B.2a	10 can be thought of as a bundle of ten ones—called a “ten.”
	1.NBT.B.2b	The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
	1.NBT.B.2c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
Use place value understanding and properties of operations to add and subtract.	1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.
	1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
	1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. Identify arithmetic patterns of 10 more and 10 less than using strategies based on place value.
	1.NBT.C.6	Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 1

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Extend the counting sequence.	<ul style="list-style-type: none"> Count by ones up to 20 <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Count by tens to 100. Represent a number of objects (between 10 and 20) with a numeral by writing and/or using layered place value cards or flip books Count forward beginning from a given number up to 20 within the known sequence (e.g., count on from 13) 	<ul style="list-style-type: none"> Count by ones up to 100 Represent a number of objects (up to 99) with a numeral by writing and/or using layered place value cards or flip books Count forward beginning from a given number up to 100 within the known sequence (e.g., count on from 23). <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Understand place value.	<ul style="list-style-type: none"> Create groups/bundles of 10 from 10, 20, and/or 30 single objects Visually represent quantities between 11 and 19 as a bundle of "ten" and the appropriate number of "ones" use manipulatives on a place value mat Label bundles with numerals of up to twenty objects grouped/bundled into tens and ones on a place value mat Compare objects bundled into one "ten" or up to 9 "ones" using the terms "greater than," "equal to," or "less than" <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Visually represent quantities up to 50 as bundles of 10 ones (as "tens"), and up to 9 ones (as "ones") using manipulatives on a place value mat Use numerals to accurately label groups of up to fifty objects grouped/bundled into tens and ones on a place value mat Compare up to 50 objects grouped into tens and ones using the terms "greater than," "equal to," or "less than" 	<ul style="list-style-type: none"> Use numerals to accurately label groups of objects (up to 99) grouped/bundled into tens and ones on a place value mat Visually represent quantities up to 99 as bundles of "tens" and the appropriate number of "ones" using manipulatives on a place value mat or drawings Compare objects grouped into tens and ones using symbols ($<$, $>$, $=$) up to 50 <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 1

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use place value understanding and properties of operations to add and subtract.	<ul style="list-style-type: none"> ♦ Add within 20 based on place value strategies use a visual representation (manipulatives, place value mats, drawings, and/or technology) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ♦ Add within 20 using equations based on place value strategies and a visual representation (e.g., manipulatives, place value mats, drawings, and/or technology) ♦ Identify “ten more” than a given two-digit number (using manipulatives, drawings, sounds, fingers, or counting by ones) 	<ul style="list-style-type: none"> ♦ Add and subtract within 5 using equations and visual representations based on place value strategies to (manipulatives, place value mats, drawings and/or technology) ♦ Identify “ten more” or “ten less” than a given two-digit number (using manipulatives bundled into tens and ones, place value mats, drawings, and/or counting on fingers) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics

DOMAIN Number and Operations in Base Ten

Grade 2

Cluster	Standards as written	
Understand place value.	2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
	Understand the following (1a and 1b) as special cases:	
	2.NBT.A.1a	100 can be thought of as a bundle of ten tens—called a “hundred.”
	2.NBT.A.1b	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
	2.NBT.A.2	Count within 1,000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting starting at any number.
	2.NBT.A.3	Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.
	2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.
Use place value understanding and properties of operations to add and subtract.	2.NBT.B.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
	2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.
	2.NBT.B.7	Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
	2.NBT.B.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
	2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. ¹

¹ Explanations may be supported by drawings or objects.

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 2

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand place value.	<ul style="list-style-type: none"> ◆ Skip-count place value units by 100 within 1000 ◆ Using place value strategies, demonstrate the concept of trading equal amounts (e.g., 10 singles = a bundle of 10; 10 bundles = a bundle of 100) ◆ Represent a number of objects (up to 200) bundled into hundreds, tens and ones with a numeral by writing, using layered place value cards and/or flip books ◆ Create groups/bundles of 100 from bundles of "tens" ◆ Compare objects grouped into one hundred or up to 9 tens and up to 9 ones using the terms "greater than," "equal to," or "less than" (e.g., 100 is greater than 8 tens and 5 ones) ◆ Show numbers in expanded form up to 99 (e.g., 11 can be shown as $10 + 1$; 12 can be shown as $10 + 2$) ◆ Use number names to accurately label groups of objects to 99 into tens and ones ◆ Add and subtract single-digit numbers <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Skip-count place value units by 10 within 100 ◆ Use numerals to accurately label groups of objects (up to 500) grouped/bundled into hundreds, tens and ones (i.e. by writing, using layered place value cards and/or flip books) ◆ Use number names to accurately label groups of objects (up to 500) grouped/bundled into hundreds, tens and ones ◆ Visually represent quantities up to 500 as bundles of 10 tens (as "hundreds"), 10 ones (as "tens"), and up to 9 ones (as "ones") using manipulatives and place value mats ◆ Express the "hundreds" digit, the "tens" digit and "ones" digit of a written numeral between 11 to 499 by using manipulatives, place value mats and/or technology ◆ Compare objects grouped into hundreds, tens and ones using the terms "greater than," "equal to," or "less than," up to 200 ◆ Show numbers in expanded form up to 200 (e.g., 111 can be shown as $100 + 10 + 1$; 125 can be shown as $100 + 20 + 5$) ◆ Add and subtract double-digit numbers 	<ul style="list-style-type: none"> ◆ Skip-count place value units by 5 within 100 ◆ Use numerals to accurately label groups of objects (up to 999) grouped/bundled into hundreds, tens and ones (i.e. by writing, using layered place value cards and/or flip books) ◆ Use number names to accurately label groups of objects (up to 999) grouped/bundled into hundreds, tens and ones ◆ Visually represent quantities up to 999 as bundles using manipulatives and place value mats ◆ Compare objects grouped into hundreds, tens and ones using symbols ($=$, $>$, $<$) up to 500 ◆ Add and subtract three-digit numbers <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 2

← **Less Complex**

More Complex →

Use place value understanding and properties of operations to add and subtract.

- ◆ Solve column addition and subtraction problems of 2 two-digit numbers that are multiples of ten using place value strategies (including manipulatives and drawings)
- ◆ Demonstrate the relationship between addition and subtraction within 10 using equations and place value materials (e.g., $5 + 3 = 8$; $8 - 3 = 5$)
- ◆ Add and subtract 100 to or from a given set of up to 900 objects

See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials

- ◆ Solve column addition and subtraction problems of two-digit numbers use place value strategies including manipulatives and drawings
- ◆ Add ten and subtract 10 to a given set of up to 100 objects
- ◆ Add up to three two-digit numbers using place value manipulatives
- ◆ Solve multi-digit column addition and subtraction problems using the standard algorithm
- ◆ Add and subtract single and/or double-digit numbers

- ◆ Solve column addition and subtraction problems of three-digit numbers use place value strategies including manipulatives and drawings
- ◆ Demonstrate the relationship between addition and subtraction within 20 using number sentences and place value materials (e.g., $5 + 12 = 17$; $17 - 5 = 12$)
- ◆ Add and subtract ten to or from a given set of up to 900 objects using strategies based on place value
- ◆ Add up to three two-digit numbers using strategies based on place value and/or place value manipulatives

Continue to address skills and concepts that approach grade-level expectations in this cluster

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Grade 3		
Cluster	Standards as written	
Use place value understanding and properties of operations to perform multi-digit arithmetic ¹ .	3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
	3.NBT.A.2	Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

¹ A range of algorithms may be used.
Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 3

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use place value understanding and properties of operations to perform multi-digit arithmetic.	<ul style="list-style-type: none"> ◆ Round whole two-digit numbers to the nearest 10 using place value materials <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Round whole three-digit numbers to the nearest 100 using place value materials 	<ul style="list-style-type: none"> ◆ Multiply one-digit numbers by a multiple of 10 (in the range of 10-90) using manipulatives, repeated addition, skip counting by tens or place value strategies ◆ Round whole three-digit numbers to the nearest 10 using place value materials <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics**DOMAIN** Number and Operations in Base Ten**Grade 4**

Cluster	Standards as written	
Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.	4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in any place represents 10 times as much as it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i>
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.
Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.	4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 4

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.	<ul style="list-style-type: none"> ◆ Compose and decompose multi-digit numbers by their place values using expanded form and base-ten materials (e.g., layered place value cards, flip books) ◆ Represent a three-digit whole number to demonstrate that the digit in tens place represents ten times what it represents in the ones place and the digit in the hundreds place represents ten times what it represents in the tens place (e.g., in number 324, the 3 represents 30 bundles of tens and the 2 represents 20 units of ones) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Compare two numbers within 100 using “more than,” “less than,” or “equal to” (e.g., place value materials) ◆ Interpret multi-digit whole numbers using base-ten materials, layered place value cards and/or flip books “express” answer ◆ Represent a four-digit whole number using place value manipulatives (e.g., in number 5,324, the 5 represents 50 hundreds, the 3 represents 30 tens and the 2 represents 20 ones) ◆ Round whole numbers to the nearest 10 using place value 	<ul style="list-style-type: none"> ◆ Show numbers in expanded form (e.g., 1,111 can be shown as $1,000 + 100 + 10 + 1$; 6,125 can be shown as $6,000 + 100 + 20 + 5$) ◆ Compare numbers within 1,000 using the symbols $>$, $<$, or $=$ ◆ Round whole numbers to the nearest 100 using place value <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.	<ul style="list-style-type: none"> ◆ Multiply up to 10×10 (e.g., using visual representations and/or manipulatives) ◆ Multiply one-digit numbers by a multiple of 100 and/or 1,000 (e.g., using manipulatives, place value strategies, or the properties of multiplication) ◆ Divide up to a two-digit number by a one-digit number without remainders (e.g., using place value materials, rectangular arrays, or area models) 	<ul style="list-style-type: none"> ◆ Multiply a one-digit number by at least a two-digit number ◆ Divide up to a three-digit number by a one-digit number without remainders (e.g., using equations and place value materials, rectangular arrays, or area models) ◆ Solve division problems up to two digits by one digit, using the relationship between multiplication and division to demonstrate that, for example, $76 \div 4 = 19$ because $19 \times 4 = 76$ ◆ Divide up to a three-digit number by a one-digit number without remainders using various strategies or methods 	<ul style="list-style-type: none"> ◆ Multiply a two-digit number by a two digit-number (e.g., using equations, an array, or area model) ◆ Divide up to a three-digit number by a one-digit number with remainders ◆ Solve three-digit by one-digit division word problems <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics**DOMAIN** Number and Operations in Base Ten**Grade 5**

Cluster	Standards as written	
Understand the place value system.	5.NBT.A.1	Recognize that in a multi-digit number, including decimals, a digit in any place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
	5.NBT.A.3	Read, write, and compare decimals to thousandths.
	5.NBT.A.3a	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$.
	5.NBT.A.3b	Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.
	5.NBT.A.4	Use place value understanding to round decimals to any place.
Perform operations with multi-digit whole numbers and with decimals to hundredths.	5.NBT.B.5	Fluently multiply multi-digit whole numbers (include two-digit \times four-digit numbers and, three-digit \times three-digit numbers) using the standard algorithm.
	5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and between multiplication and division; relate the strategy to a written method and explain the reasoning used.

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 5

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand the place value system.	<ul style="list-style-type: none"> Express decimals to the hundredths (e.g., using place value materials, flip books) Compare decimals to the tenths using symbols ($=$, $>$, $<$) (e.g., flip books and/or manipulatives) Connect money to decimals by rounding up to the nearest dollar (e.g., \$2.57 becomes \$3) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Express decimals to the thousandths (e.g., using place value materials, flip books) Compare decimals to the hundredths using symbols ($=$, $>$, $<$) (e.g., using place value materials, flip books and/or manipulatives) Express decimals to the hundredths using expanded form Show that in a three-digit whole number, a digit in one place represents $1/10$ what it represents in the place to the left (e.g., use place value materials to show that the 5 in 356 is $1/10$ the value of the 5 in 514) Connect money to decimals by rounding up to the nearest dime (e.g., \$2.57 becomes \$2.60) 	<ul style="list-style-type: none"> Write decimals to the hundredths (e.g., using place value materials, flip books) Write decimals to the hundredths using expanded form Rounds decimals to the nearest tenth Use whole number exponents to denote powers of ten (e.g., show that $10 \times 10 \times 10 = 1,000$ and 10^3) Multiply multi-digit whole numbers <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Number and Operations in Base Ten Standards in Grade 5

← Less Complex

More Complex →

<p>Perform operations with multi-digit whole numbers and with decimals to hundredths.</p>	<ul style="list-style-type: none"> ◆ Add decimals to the tenths using various strategies ◆ Subtract decimals to the tenths using various strategies <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Divide up to a three-digit number by a two-digit number without remainders (e.g., using place value materials, rectangular arrays or area models) ◆ Multiply a whole number by a decimal in the tenths using estimation (e.g., 6×0.19 is close to $6 \times \\$0.20$ which is \$1.20 my number will be one whole and a decimal.) ◆ Multiply a whole number by a decimal in the tenths using number lines ◆ Multiply a whole number by a decimal in the tenths using repeated addition ◆ Multiply a whole number by a decimal in the tenths using coins ◆ Multiply a whole number by a decimal in the tenths (e.g., using a hundreds grid, I can shade 19 squares 6 times to show on whole grid and 14 hundredths or 1.14) ◆ Divide up to a four-digit number by a two-digit number with remainders using equations ◆ Divide a number that includes a decimal to the hundredths by a decimal 	<ul style="list-style-type: none"> ◆ Divide up to a three-digit number by a two-digit number with remainders (e.g., using place value materials, rectangular arrays, or area models) ◆ Add decimals to hundredths (e.g., place value materials, concrete models, drawings or strategies based on properties of operations) ◆ Subtract decimals to hundredths (e.g., using place value materials, concrete models, drawings, or strategies based on properties of operations) ◆ Divide a number that includes a decimal to tenths by a whole number using estimation based on properties of operations (e.g., if $7.6 \div 4$ is close to $8 \div 4$, my answer will be close to 2) ◆ Divide a number that includes a decimal to tenths by a whole number using repeated subtraction based on properties of operations ◆ Divide a number that includes a decimal by a whole number using coins based on properties of operations (e.g., share \$7.60 with 4 people; each person will get 1 dollar and 9 dimes. My answer is 1.90) ◆ Divide a number that includes a decimal to tenths by a whole number <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
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Number and Operations – Fractions

	Standards	Entry Points	Access Skills
3	Page 59	Pages 60 – 61	Pages 60 – 62
4	Pages 63 – 64	Page 65 – 67	
5	Pages 68 – 69	Page 70 – 72	

CONTENT AREA Mathematics
DOMAIN Number and Operations–Fractions

Grade 3

Cluster	Standards as written	
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.	3.NF.A.1	Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole (a single unit) is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.
	3.NF.A.2a	Represent a unit fraction, $\frac{1}{b}$, on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the fraction $\frac{1}{b}$ is located $\frac{1}{b}$ of a whole unit from 0 on the number line.
	3.NF.A.2b	Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.
	3.NF.A.3	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.
	3.NF.A.3a	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
	3.NF.A.3b	Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
	3.NF.A.3c	Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. <i>For example, express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.</i>
	3.NF.A.3d	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

ENTRY POINTS and ACCESS SKILLS for Number and Operations–Fractions Standards in Grade 3

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>
	<u>The student will:</u>	<u>The student will:</u>
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.	<ul style="list-style-type: none"> ◆ Respond to materials that demonstrate objects that can be divided into equal parts ◆ Track materials that demonstrate that objects can be divided into equal parts ◆ Shift focus from materials that demonstrate that objects can be divided into equal parts ◆ Grasp materials that demonstrate that objects can be divided into equal parts ◆ Use two hands to hold materials that demonstrate that objects can be divided into equal parts ◆ Release materials that demonstrate that objects can be divided into equal parts ◆ Move materials that demonstrate that objects can be divided into equal parts 	<ul style="list-style-type: none"> ◆ Explain what the numerator and denominator of a fraction represent (e.g., in $\frac{2}{4}$, the 4 tells you how many parts the whole is divided into and 2 is the number of parts you have and use a drawing to illustrate) ◆ Identify concepts of whole and $\frac{1}{2}$ using manipulatives and/or familiar objects (e.g., using sets of objects or shapes with shaded parts, identify $\frac{1}{2}$ and whole) ◆ Partition a whole into $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ equal parts using visual models, number lines, or manipulatives (e.g., given a rectangle, draw lines to divide it into 3 equal parts representing $\frac{1}{3}$) ◆ Compare fractions of the same whole to determine which is greater (e.g., show a number line with points at $\frac{1}{4}$ and $\frac{3}{4}$ and ask which is greater)
		<ul style="list-style-type: none"> ◆ Compare visual representations of fractions using the terms "greater than," "less than," or "equal to" (e.g., verbalize that $\frac{1}{2}$ is greater than $\frac{1}{4}$) ◆ Match visual representations of simple fractions to the name of the fraction (e.g., given visual fraction models with sections already shaded in, identify amounts such as $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$) ◆ Compare parts of a whole (quarters, thirds, halves) to determine relative size of each ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) using manipulatives or visual models (e.g., use manipulatives to show that $\frac{1}{2} > \frac{1}{3}$) ◆ Label unit fractions* on a number line (e.g., given a number line labeled with 0 and 1 and divided into 6 equal parts, plot and label a point at $\frac{1}{6}$)
		<ul style="list-style-type: none"> ◆ Identify parts of a whole using visual fraction models (e.g., using shapes divided into equal parts and having one part shaded in, identify $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$) ◆ Divide a number line into equal parts and label points (e.g., mark off a number line labeled with 0 and 1 into 6 equal sections and label each point with $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, etc.) ◆ Record results of the comparisons of two fractions with like denominators or like numerators using symbols (e.g., use the symbols $<$, $=$, or $>$ to write a comparison of $\frac{1}{4}$ and $\frac{3}{4}$, or $\frac{2}{4}$ and $\frac{2}{8}$) ◆ Create a visual representation of simple fractions (e.g., divide a shape into 4 equal parts and shade in $\frac{3}{4}$ recognizing that the parts do not need to be touching)

ENTRY POINTS and ACCESS SKILLS for Number and Operations–Fractions Standards in Grade 3

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>
	<u>The student will:</u>	<u>The student will:</u>
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8. (continued)	<ul style="list-style-type: none"> ♦ Orient materials that demonstrate that objects can be divided into equal parts ♦ Locate objects partially hidden or out of sight (e.g., remove barrier to expose part that when added to object equals the whole object) ♦ Turn device on/off to participate in an activity on fractions (e.g., turn on voice-generating device) to comment on fraction activity ♦ Imitate action required to divide object ♦ Initiate cause-and-effect response (e.g., turn on technology tool) to activate fraction activity ♦ Sustain activity through response in a fraction-based activity ♦ Gain attention (e.g., request a turn) with fraction materials ♦ Make a request in a fraction based activity 	<ul style="list-style-type: none"> ♦ Answer questions about fractions (e.g., Show a shaded figure that represents $\frac{3}{6}$ and answer "Does this show the fraction $\frac{3}{6}$ or $\frac{3}{4}$?") ♦ Match a fraction represented visually to the fraction represented numerically or verbally (e.g., a fraction model) ♦ Create equivalent fractions using manipulatives (e.g., using two equivalent wholes made up of manipulatives, show that $\frac{2}{4} = \frac{1}{2}$ with the appropriate shapes)
		<ul style="list-style-type: none"> ♦ Partition a number line labeled with 0 and 1 into 2, 4, or 8 equal parts (e.g., draw hash marks on the number line to divide the length from 0 to 1 into 4 equal parts) ♦ Order simple fractions by plotting and labeling points on two number lines that have already been divided into equal parts (e.g., plot points at $\frac{5}{8}$ on one line and $\frac{1}{2}$ on another line to show that $\frac{5}{8} > \frac{1}{2}$) ♦ Express whole numbers as fractions using models and show they are equal to 1 (e.g., divide two equivalent shapes into 4 equal parts and 6 equal parts to show that $\frac{4}{4} = \frac{6}{6} = 1$ as long as the wholes are equal in size) ♦ Match a visual representation of a fraction to a fractional number line (e.g., match a shape with $\frac{1}{4}$ shaded to a number line with a point at $\frac{1}{4}$)
		<ul style="list-style-type: none"> ♦ Order simple fractions on a number line that has already been divided into equal parts (e.g., plot points at $\frac{2}{8}$ and $\frac{1}{2}$ on a number line divided into 8 sections to show that $\frac{2}{8} < \frac{1}{2}$) ♦ Label points with simple fractions on a number line (e.g., label given points at $\frac{1}{6}$, $\frac{3}{6}$, $\frac{5}{6}$ on a number line) ♦ Determine the number of unit fractions* in a whole by using same sized pieces to create a whole (e.g., If you need 3 pieces to make a whole then each piece represents the unit fraction* $\frac{1}{3}$) ♦ Express whole numbers as fractions and fractions as whole numbers using models and show they are equivalent (e.g., given two equivalent shapes divided into 4 equal parts, show that $\frac{4}{4} = 1$ and $\frac{8}{4} = 2$)
		<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>* Unit Fraction: a fraction with a numerator of one</p> </div>
		<p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

**ACCESS SKILLS (continued) for
Number and Operations–Fractions Standards in Grade 3**

Less Complex

More Complex

ACCESS SKILLS
The student will:

ENTRY POINTS
The student will:

Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.
(continued)

- ♦ Choose from an array of two in a fraction-based activity (e.g., choose materials to be divided into equal parts)
- ♦ Attend visually, aurally, or tactilely to materials that demonstrate objects that can be divided into equal parts

CONTENT AREA Mathematics
DOMAIN Number and Operations–Fractions

Grade 4

Cluster	Standards as written	
Extend understanding of fraction equivalence and ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	4.NF.A.1	Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.
	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	4.NF.B.3	Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.
	4.NF.B.3a	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (The whole can be a set of objects.).
	4.NF.B.3b	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2 \frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.
	4.NF.B.3c	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
	4.NF.B.3d	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using drawings or visual fraction models and equations to represent the problem.
	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
	4.NF.B.4a	Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. <i>For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.</i>
	4.NF.B.4b	Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. <i>For example, use a visual fraction model to express $3 \square (2\frac{5}{8})$ as $6 \square (\frac{15}{8})$, recognizing this product as $6\frac{5}{8}$. (In general, $n \square (\frac{a}{b}) = (n \square a)\frac{1}{b}$.)</i>

	4.NF.B.4c	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i>
Understand decimal notation for fractions, and compare decimal fractions.	4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. <i>For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.</i>
	4.NF.C.6	Use decimal notation to represent fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i>
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 4

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Extend understanding of fraction equivalence and ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	<ul style="list-style-type: none"> ◆ Distinguish between equal and non-equal parts of a whole (e.g., compare unit fractions* or fractions with like denominators but unlike numerators using models or number lines) ◆ Identify a fraction $1/b$ as the quantity formed by one part when a whole is partitioned into b equal parts (e.g., given several shapes with unit fractions* shaded in, identify the parts of the whole as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.) ◆ Demonstrate fractions equivalent to $\frac{1}{2}$ using fraction models, manipulatives and/or technology (e.g., show that $\frac{2}{4} = \frac{1}{2}$ on a number line) ◆ Compare two fractions with like denominators by comparing their relative size (e.g., fraction models) ◆ Identify which of two fractions represents a larger part of a whole using fraction models or manipulatives (e.g., given two number lines marked from 0 to 1 with points plotted for $\frac{4}{5}$ and $\frac{3}{8}$, determine that $\frac{4}{5}$ is larger because it is more of the whole because it is to the right of $\frac{3}{8}$) ◆ Compare two fractions with unlike denominators by comparing their relative size using fraction models 	<ul style="list-style-type: none"> ◆ Identify equivalent fractions using fraction models, manipulatives, and/or technology (e.g., given several rectangles already shaded in with different fraction amounts, show that $\frac{2}{6} = \frac{1}{3}$) ◆ Compare two fractions with like denominators, represented numerically, using $>$, $<$, or $=$ (e.g., $\frac{5}{6} > \frac{3}{6}$) ◆ Determine which of two fractions with like denominators represents a larger part of a whole by representing the fractions with fraction models or manipulatives (e.g., given two congruent rectangles divided into eighths, shading in one to represent $\frac{3}{8}$ and another to represent $\frac{7}{8}$, and then identifying which is more of the whole) 	<ul style="list-style-type: none"> ◆ Generate multiple pairs of equivalent fractions using fraction models, manipulatives and/or technology (e.g. show that $\frac{2}{4} = \frac{3}{6}$ by drawing two congruent rectangles with one divided into fourths and one divided into sixths and shading in $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ to show they are equivalent areas) ◆ Compare two fractions with unlike denominators by demonstrating which is greater or less than the benchmark of $\frac{1}{2}$ using fraction models, manipulatives or technology (e.g., showing on two number lines that $\frac{2}{8} < \frac{3}{4}$ because $\frac{2}{8}$ is to the left of $\frac{1}{2}$ on the number line and $\frac{3}{4}$ is to the right of $\frac{1}{2}$) ◆ Demonstrate, using fraction models or manipulatives, that the whole is equal to the sum of the partitioned parts (e.g., $\frac{4}{4} = 1$; $1 = \frac{8}{8}$) ◆ Compare visual models of fractions with unlike denominators using symbols ($<$, $>$, or $=$) (e.g., given two models, one of $\frac{4}{5}$ and one with $\frac{3}{10}$, determine which is greater and then write $\frac{4}{5} > \frac{3}{10}$)

Continue to address skills and concepts that approach grade-level expectations in this cluster

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 4

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	<ul style="list-style-type: none"> ◆ Add unit fractions* with like denominators with only two addends using fraction models (e.g., using a number line or shapes, show that $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ or 1) ◆ Add simple fractions using visual models, manipulatives, or technology (e.g., showing that two halves equal a whole or two fourths equal a half) ◆ Subtract simple fractions using visual models, manipulatives or technology (e.g. using manipulatives, show $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add unit fractions* with like denominators with more than two addends using both fraction models and equations (e.g., using a number line or shapes, show that $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ or $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ and then write the equation) ◆ Add and subtract fractions with like denominators using visual fraction models (e.g. use a rectangle divided into 12 equal parts to solve $\frac{2}{12} + \frac{3}{12}$ by shading 2 parts and then 3 parts to find the total number of twelfths) ◆ Multiply a fraction by a whole number using visual models and repeated addition (e.g., showing that $\frac{1}{4} \times 3 = \frac{3}{4}$ and $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$) 	<ul style="list-style-type: none"> ◆ Decompose a fraction into a sum of unit fractions* with the same denominator one way (e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$) ◆ Subtract fractions with like denominators using both fraction models and equations (e.g., show $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$ using a number line marked with eighths to “hop” three eighths to the left from $\frac{5}{8}$ to end up at $\frac{2}{8}$ and then write the equation) ◆ Add and subtract fractions and mixed numbers with like denominators when mixed numbers do not need to be re-written as fractions (e.g. $4\frac{3}{4} + \frac{1}{4} = 5$, $2\frac{4}{5} - \frac{3}{5} = 2\frac{1}{5}$) ◆ Solve word problems involving addition and subtraction of no more than two fractions with like denominators using fraction models, manipulatives, or technology (e.g., Jamie has $\frac{1}{8}$ cup of apple juice and $\frac{4}{8}$ cup of pineapple juice. What is the total amount of juice, in cups, that Jamie has in all?) ◆ Represent a mixed number as an equivalent fraction using fraction models, manipulatives, or technology, given the written form (e.g., show that $1\frac{1}{2} = \frac{3}{2}$)

*** Unit Fraction:**
a fraction with a numerator of one

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 4

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (continued)			<ul style="list-style-type: none"> ◆ Multiply a non-unit fraction by a whole number using visual models, manipulatives, or technology (e.g., $3 \times \frac{2}{4} = \frac{2}{4} + \frac{2}{4} + \frac{2}{4}$ also equals 3 groups of $\frac{2}{4}$ and each $\frac{2}{4} = \frac{1}{4} + \frac{1}{4}$) ◆ Solve a multiplication word problem involving multiplying a fraction by a whole number using visual models, manipulatives, or technology (e.g., Danny rode his bike $\frac{3}{8}$ mile each day in the summer. What is the total distance Danny rode over 5 days? $5 \times \frac{3}{8} = \frac{15}{8}$ or $1 \frac{7}{8}$ miles) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Understand decimal notation for fractions, and compare decimal fractions.	<ul style="list-style-type: none"> ◆ Order decimals on a number line (e.g., given a number line labeled with tenths in decimals, plot points at 0.4, 0.7, and 0.8 or given a number line labeled with hundredths in decimals, plot points at 0.62, 0.64, and 0.68) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Show that a fraction with a denominator of ten is equivalent to a fraction with a denominator of 100 by using visual models, manipulatives, or technology (e.g., using base-ten blocks, show that $\frac{5}{10} = \frac{50}{100}$) ◆ Compare two decimals up to the hundredths by reasoning about their size using symbols ($<$, $>$, $=$) or visual model 	<ul style="list-style-type: none"> ◆ Express a fraction with a denominator of ten as an equivalent fraction with a denominator of 100 (e.g., $\frac{3}{10} = \frac{30}{100}$ or $\frac{60}{100} = \frac{6}{10}$) ◆ Use decimal notation for fractions with denominators of ten. (e.g., $\frac{2}{10} = 0.2$) ◆ Compare two decimals to the tenths by reasoning about their size using symbols ($<$, $>$, or $=$) or visual model (e.g., use a number line to show that $0.65 > 0.40$ because 0.40 is to the left of 0.65 and write the inequality) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics

DOMAIN Number and Operations–Fractions

Grade 5

Cluster	Standards as written	
Use equivalent fractions as a strategy to add and subtract fractions.	5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. <i>For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)</i>
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. <i>For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.</i>
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	5.NF.B.3	Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when three wholes are shared equally among four people each person has a share of size $\frac{3}{4}$. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i>
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
	5.NF.B.4a	Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. <i>For example, use a visual fraction model and/or area model to show $(\frac{2}{3}) \square 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \square (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \square (\frac{c}{d}) = \frac{ac}{bd}$.)</i>
	5.NF.B.4b	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
	5.NF.B.5a	Interpret multiplication as scaling (resizing), by: comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. <i>For example, without multiplying tell which number is greater: 225 or $\frac{3}{4} \times 225$; $\frac{11}{50}$ or $\frac{3}{2} \times \frac{11}{50}$?</i>

to multiply and divide fractions. (continued)	5.NF.B.5b	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1
	5.NF.B.6	Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. ¹
	5.NF.B.7a	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. <i>For example, create a story context for $(\frac{1}{3}) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = \frac{1}{12}$ because $(\frac{1}{12}) \times 4 = \frac{1}{3}$.</i>
	5.NF.B.7b	Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, create a story context for $4 \div (\frac{1}{5})$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$ because $20 \times (\frac{1}{5}) = 4$.</i>
	5.NF.B.7c	Solve real-world problems involving division of unit fractions by non-zero whole number and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if three people share $\frac{1}{2}$ lb. of chocolate equally? How many $\frac{1}{3}$-cup servings are in two cups of raisins?</i>

¹ Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 5

← **Less Complex**

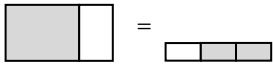
More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use equivalent fractions as a strategy to add and subtract fractions.	<ul style="list-style-type: none"> ◆ Add fractions with like denominators creating sums greater than/or equal to one (e.g., $7/10 + 4/10 = 11/10$) ◆ Subtract fractions with like denominators creating differences less than one (e.g., $6/8 - 3/8 = 3/8$) ◆ Identify two equivalent fractions with unlike denominators that are represented by fraction models (e.g. given two fraction models referring to the same whole, show that they are $3/4$ and $6/8$ and are equivalent) ◆ Identify visual fraction models that represent mixed numbers (e.g., identify a fraction model consisting of 2 rectangles with 1 and $3/4$ shaded in as representing $1\frac{3}{4}$) ◆ Compare two fractions with like numerators or like denominators by reasoning about their size (e.g., explain that $3/8 < 3/6$ because eighths are a smaller size than sixths so if you have 3 sixths of a pizza you have more than if you have 3 eighths of the same size pizza) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add unit fractions* with unlike denominators, by using manipulatives or technology to create equivalent fractions with like denominators (e.g., $1/6 + 1/4 = 2/12 + 3/12 = 5/12$) ◆ Subtract unit fractions* with unlike denominators, by using manipulatives or technology to create equivalent fractions with like denominators (e.g., $1/4 - 1/8 = 2/8 - 1/8 = 1/8$) ◆ Represent mixed numbers with fraction models (e.g., draw a fraction model to represent $3/2$ and show that $3/2 = 1\frac{1}{2}$) ◆ Represent word problems with fractions with like denominators (e.g., represent a word problem that requires the student to add $1/8 + 5/8$ with fraction models referring to the same whole) ◆ Compare two fractions with different numerators and different denominators (e.g., $1/2 < 5/8$ using technology, or $1/2$ is equal to $6/12$ using a fraction model) ◆ Use benchmark fractions to compare fractions with like denominators using visual fraction models, manipulatives, or technology (e.g. know that $7/10 > 1/2$ and $4/9 < 1/2$ so therefore $7/10 > 4/9$) 	<ul style="list-style-type: none"> ◆ Add fractions with unlike denominators, creating sums less than one by using manipulatives or technology to create equivalent fractions with like denominators (e.g., $2/5 + 1/3 = 6/15 + 5/15 = 11/15$) ◆ Subtract fractions with unlike denominators, creating differences less than one by using manipulatives or technology to create equivalent fractions with like denominators (e.g., $4/5 - 2/3 = 12/15 - 10/15 = 2/15$) ◆ Add and subtract mixed numbers with like denominators using manipulatives or technology (e.g., $1\frac{1}{3} + 2\frac{2}{3} = 3\frac{3}{3} = 4$ or $2\frac{1}{4} - 1\frac{3}{4} = 9/4 - 7/4 = 2/4$) ◆ Solve word problems involving addition or subtraction of fractions with like denominators using manipulatives or technology (e.g., draw fraction models to represent the fractions and the solution) ◆ Estimate sums or differences of fractions with like denominators, use benchmark fractions and number sense (e.g. knowing that $3/5 + 4/5 = 7/10$ is false because $3/5$ and $4/5$ are both greater than $1/2$ so the sum must be greater than 1 and $7/10$ is less than 1)

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 5

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use equivalent fractions as a strategy to add and subtract fractions. (continued)		<ul style="list-style-type: none"> Show valid comparisons of fractions that refer to the same whole (e.g., given two pictures of wholes divided into 6ths and 12ths, show that $\frac{2}{6}$ is equal to $\frac{4}{12}$) 	<ul style="list-style-type: none"> Show that comparisons are valid only if the fractions refer to the same whole (e.g., given pictures of pairs of fractions, some with different size wholes and some with the same size wholes, be able to choose the correct comparisons: <div style="text-align: center;">  </div> <p>is not true because the wholes are not the same size)</p> <p style="text-align: right;"><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	<ul style="list-style-type: none"> Show/express that a unit fraction* is represented by the division of the numerator by the denominator (e.g., show that $\frac{1}{5}$ means dividing one whole into 5 equal parts) Write a multiplication problem involving a whole number and a fraction as a repeated addition problem (e.g., $4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$) Show that multiplying a fraction by a fraction is similar to creating a model of the first fraction, then scaling each part by the other fraction (e.g. $\frac{1}{4} \times \frac{1}{2}$ can be modeled by describing or drawing a whole rectangle divided into 4 equal parts, and then each of the 4 parts ($\frac{1}{4}$) is divided into 2 equal parts so that each part is now $\frac{1}{8}$ of the whole rectangle) 	<ul style="list-style-type: none"> Match fractions with their equivalent division expressions (e.g. $\frac{3}{8} = 3 \div 8$, not $8 \div 3$ nor 3×8) Represent connections between fractions and division with the use of visual models, manipulatives or technology (e.g., show $\frac{8}{4}$ can be represented as 8 candy bars into 4 groups results in each group getting 2 candy bars) Multiply a whole number by a unit fraction* using a number line marked from 0 to 1 (e.g. use a number line labeled with $\frac{1}{4}$'s to find $3 \times \frac{1}{4} = \frac{3}{4}$) Compare products of fractions and whole numbers based on the multiple using a visual fraction model (e.g., is $5 \times \frac{2}{5}$ greater or less than $5 \times \frac{4}{5}$ or is $\frac{3}{5} \times \frac{1}{2}$ greater or less than $\frac{7}{6} \times \frac{1}{2}$) 	<ul style="list-style-type: none"> Identify division in a real-world problem as a fraction (e.g., write "5 cookies divided equally by 10 people means each person gets $\frac{5}{10}$ or $\frac{1}{2}$ of a cookie") Multiply fractions by fractions using manipulatives, visual models and/or technology (e.g., $\frac{2}{4} \times \frac{4}{5} = \frac{8}{20}$) Multiply a whole number by a fraction less than 1 using visual models or manipulatives (e.g., show $\frac{2}{3} \times 4 = \frac{8}{3}$ by using 4 shapes that are determined to be $\frac{2}{3}$ of a whole or by defining a whole and using 2 one-third shapes as the $\frac{2}{3}$ and compiling 4 sets of the $\frac{2}{3}$)

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 5

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (continued)	<ul style="list-style-type: none"> ◆ Solve real-world problems involving division of a whole into equal parts (e.g., divide a whole candy bar to share with four friends) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Show that dividing a fraction by a whole number will give a quotient that is less than the original using fraction models or manipulatives (e.g. $\frac{1}{2} \div 3$ will be equal to a fraction that is less than $\frac{1}{2}$) ◆ Show that dividing a whole number by a fraction will give a quotient that is greater than the original whole number using fraction models or manipulatives (e.g., $3 \div \frac{1}{2}$ will be greater than 3) <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>* Unit Fraction: a fraction with a numerator of one</p> </div>	<ul style="list-style-type: none"> ◆ Compare the size of a product to the size of one factor (with one factor being a fraction and one factor a whole number) without performing the multiplication (e.g., know that $\frac{2}{3} \times 4 < 4$ because $\frac{2}{3} < 1$ so the product must be less than 1×4) ◆ Solve real-world problems by multiplying fractions or mixed numbers using manipulatives, visual models, and/or technology (e.g., solve “There are 4 students who need paint for their art project. Each student needs $1\frac{3}{4}$ gallons of paint. What is the total amount of paint, in gallons, needed by these four students?” by using a visual fraction model to show $1\frac{3}{4}$ four times to find solution) ◆ Solve real-world problems by dividing a whole number by a unit fraction* or a unit fraction by a whole number (e.g., 3 pizzas are each divided into fourths so there are 12 pieces, $3 \div \frac{1}{4} = 12$) ◆ Connect division and multiplication of fractions (e.g., know that $5 \div \frac{1}{2} = 10$ because $10 \times \frac{1}{2} = 5$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

The Number System

	Standards	Entry Points	Access Skills
6	Pages 74 – 75	Pages 76 – 79	Pages 76 – 80
7	Page 81	Page 82	
8	Page 83	Page 84	

CONTENT AREA Mathematics
DOMAIN The Number System

Grade 6

Cluster	Standards as written	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. In general, $(a/b) \div (c/d) = ad/bc$. How much chocolate will each person get if three people share $1/2$ lb. of chocolate equally? How many $3/4$-cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mile and area $1/2$ square mile?</i>
Compute fluently with multi-digit numbers and find common factors and multiples.	6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.
	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
	6.NS.B.4	Use prime factorization to find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two relatively prime numbers. <i>For example, express $36 + 8$ as $4(9 + 2)$.</i>
Apply and extend previous understandings of numbers to the system of rational numbers.	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, and positive/negative electric charge). Use positive and negative numbers (whole numbers, fractions, and decimals) to represent quantities in real-world contexts, explaining the meaning of zero in each situation.
	6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
	6.NS.C.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that zero is its own opposite.
	6.NS.C.6b	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7	Understand ordering and absolute value of rational numbers.

Apply and extend previous understandings of numbers to the system of rational numbers. (continued)	6.NS.C.7a	Interpret statements of inequality as statements about the relative positions of two numbers on a number line diagram. <i>For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.</i>
	6.NS.C.7b	Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write $-30^{\circ}\text{C} > -70^{\circ}\text{C}$ to express the fact that -30°C is warmer than -70°C.</i>
	6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.</i>
	6.NS.C.7d	Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.</i>
	6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

ENTRY POINTS and ACCESS SKILLS for The Number System Standards in Grade 6

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	<ul style="list-style-type: none">◆ Respond to materials that demonstrate objects that can be divided into equal parts◆ Track materials that demonstrate that objects can be divided into equal parts◆ Shift focus from materials that demonstrate that objects can be divided into equal parts◆ Grasp materials that demonstrate that objects can be divided into equal parts◆ Use two hands to hold materials that demonstrate that objects can be divided into equal parts◆ Release materials that demonstrate that objects can be divided into equal parts◆ Move materials that demonstrate that objects can be divided into equal parts	<ul style="list-style-type: none">◆ Demonstrate, using manipulatives, that the whole is equal to the sum of partitioned parts (e.g., show that four equal quarter-sized parts of an object together comprise the entire object)◆ Identify parts of a whole using concrete objects (e.g., distinguish one quarter of an object from one half of the object)◆ Match visual representations of simple fractions to the fraction itself (e.g., one third of a pie as $\frac{1}{3}$)◆ Match improper fractions to equivalent mixed numbers or mixed numbers to equivalent improper fractions (e.g., from side-by-side lists)◆ Use manipulatives and strategies to show repeated division (e.g., paper folding)	<ul style="list-style-type: none">◆ Identify parts of a whole using visual fraction models (e.g., $\frac{1}{2}, \frac{2}{3}, \frac{7}{8}$, etc.)◆ Convert an improper fraction to a mixed number or a mixed number to an improper fraction (e.g., equate $\frac{5}{4}$ and $1\frac{1}{4}$)◆ Multiply a fraction by a whole number (e.g., $4 \times \frac{2}{3}, \frac{3}{5} \times 18$, etc.)◆ Divide a whole number by a fraction or a fraction by a whole number (e.g., $12 \div \frac{1}{5}, \frac{2}{7} \div 2$, etc.)◆ Create visual representations of simple fractions to the fraction itself (e.g., show one third of a pie as $\frac{1}{3}$)	<ul style="list-style-type: none">◆ Multiply a fraction by a fraction (e.g., $\frac{7}{8} \times \frac{4}{7}, \frac{1}{2} \times \frac{6}{5}$, etc.)◆ Divide two fractions with the same denominator (e.g., $\frac{2}{3} \div \frac{5}{3}, \frac{6}{7} \div \frac{8}{7}$, etc.)◆ Divide two fractions with the same numerator (e.g., $\frac{4}{3} \div \frac{9}{2}, \frac{9}{2} \div \frac{9}{4}$, etc.)◆ Solve word problems involving multiplication and division of fractions (e.g., Graham has $\frac{2}{3}$ cup of soda and Lucy has half as much soda as Graham) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS and ACCESS SKILLS for The Number System Standards in Grade 6

← Less Complex

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	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Compute fluently with multi-digit numbers and find common factors and multiples.	<ul style="list-style-type: none"> Respond to materials as they are counted Shift focus from materials to speaker counting materials Grasp materials as they are counted Release materials as they are counted Give materials as they are counted Move objects as they are counted Orient objects as they are counted (e.g., turn flowerpots upright) Manipulate objects with two hands as they are counted Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted Use one object to act on another as objects are counted (e.g., use a pointer to tap) Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall) 	<ul style="list-style-type: none"> Describe real-world quantities as greater than or less than zero (e.g., elevation, temperature, etc.) Add and/or subtract one-digit whole numbers to/from two-digit whole numbers (e.g., $6 + 11$, $81 - 8$, etc.) Multiply and divide one- and two-digit whole numbers (e.g., 5×14, $68 \div 4$, etc.) Multiply multi-digit numbers by one-digit numbers using manipulatives (e.g., illustrate 3×12 by combining three groups of twelve objects) Divide two-digit numbers by one-digit numbers using manipulatives (e.g., illustrate $15 \div 3$ by separating fifteen objects into three groups of five) Represent repeated addition problems using concrete manipulatives (e.g., $7 + 7 + 7$ can be represented by three groups of seven) Identify numbers that are multiples of 2 (e.g., from a list of numbers) Multiply one-digit number by a one-digit number 	<ul style="list-style-type: none"> Multiply a whole number by a decimal (e.g., 0.7×13, 16×2.1, etc.) Multiply a three-digit number by a one-digit number (e.g., 6×101, 3×313, etc.) Divide a two-digit number by a one-digit number (e.g., without remainders) Add and subtract numbers including decimals to tenths (e.g., $9.5 + 4.7$, $49.1 - 6.0$, etc.) Identify numbers that are multiples of 2 or 3 (e.g., from a list of numbers) Identify numbers within 50 that have a common factor (e.g., 9 and 33 have a common factor of 3, and 33 and 44 have a common factor of 11) Divide numbers including decimals to tenths (e.g., $30.3 \div 3.0$, $6.4 \div 12.8$, etc.) Factorize numbers to 50 (e.g., the factors of 28 are 1, 2, 4, 7, 14, 28) 	<ul style="list-style-type: none"> Multiply two decimal numbers (e.g., 21.4×1.3, 6.7×6.22, etc.) Multiply multi-digit numbers (e.g., 124×71, 88×88, etc.) Divide three-digit numbers by one-digit numbers (e.g., without remainders) Add and subtract numbers including decimals to hundredths (e.g., $71.62 + 22.82$, $150.01 - 77.8$, etc.) Identify numbers within 100 that have a common factor (e.g., 91 and 13 have a common factor of 13, but 33 and 49 have no common factors) Demonstrate the distributive property by finding a common factor (e.g., show $36 + 48$ as $12(3 + 4)$) Divide numbers including decimals to hundredths (e.g., $30.36 \div 3.0$, $6.40 \div 12.8$, etc.) Factorize numbers to 100 (e.g., the factors of 80 are 1, 2, 4, 5, 8, 10, 16, 20, 40, 80, but the factors of 83 are 1 and 83)

ENTRY POINTS and ACCESS SKILLS for The Number System Standards in Grade 6

Less Complex

More Complex

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Compute fluently with multi-digit numbers and find common factors and multiples. (continued)	<ul style="list-style-type: none"> Construct using materials that have been counted in sequence (e.g., tower of blocks) Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) Imitate action in counting sequence activity Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) Sustain counting sequence activity through response Gain attention in counting sequence activity Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game) Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) 	<ul style="list-style-type: none"> Demonstrate a variety of number combinations that when added equal a whole number using manipulatives (e.g., twelve objects can be separated into groups of 4 and 8 objects, 2 and 10 objects, etc.) Identify equivalent forms of common decimals and fractions less than 1 (e.g., $0.5 = \frac{1}{2}$) 	<ul style="list-style-type: none"> Use factors of to show different multiplication expressions to represent a number (e.g., 20 can be represented by 1×20, 2×10, 4×5) Add and/or subtract whole digit numbers <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS and ACCESS SKILLS for The Number System Standards in Grade 6

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> The student will:	<u>ENTRY POINTS</u> The student will:	<u>ENTRY POINTS</u> The student will:	<u>ENTRY POINTS</u> The student will:
Apply and extend previous understandings of numbers to the system of rational numbers.	<ul style="list-style-type: none"> ◆ Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted ◆ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ◆ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall) ◆ Gain attention in counting sequence activity ◆ Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game) ◆ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) 	<ul style="list-style-type: none"> ◆ Locate numbers on a horizontal number line including positive and negative integers and zero (e.g., given a calibrated number line and a point of reference, locate -5, 0, 7, etc.) ◆ Locate the negative number on a number line that is an equal distance from zero as its opposite (e.g., locate -6 on a number line given the locations of 0 and 6) ◆ Use inequality symbols to compare negative numbers (e.g., choose < or > when given two numbers to compare) ◆ Determine the absolute value of positive and negative numbers (e.g., -2 is 2 because -2 is 2 units from zero) ◆ Represent a real-life negative quantity using a vertical or horizontal number line (e.g., degrees below zero, meters below sea level, etc.) 	<ul style="list-style-type: none"> ◆ Locate fractions between 0 and 1 on a number line (e.g., given a calibrated number line and reference points 0 and 1, locate $\frac{1}{2}$, $\frac{3}{5}$, $\frac{5}{8}$, etc.) ◆ Plot points in the first quadrant on a coordinate grid (e.g., locate the points (2,3), (5,1), etc.) ◆ Determine how many units separate two points on a number line that includes positive and negative numbers (e.g., find the distance between -5 and 7) ◆ Compare absolute values using a number line (e.g., $-7 > 3$ because -7 is further away from zero than 3 and $-5 = 5$ because they are both 5 units from zero) 	<ul style="list-style-type: none"> ◆ Locate positive numbers including fractions and mixed numbers on a number line (e.g., given a calibrated number line and reference point(s), locate $\frac{1}{4}$, 3, $2\frac{1}{2}$, etc.) ◆ Locate rational numbers on a number line (e.g., including fractions and negatives) ◆ Plot points on a coordinate grid (e.g., locate the points (-3,10), (0,2), (-1,-1), etc.) ◆ Determine the coordinates of points plotted on a coordinate grid (e.g., from any quadrant) ◆ Add and subtract the absolute values of rational numbers (e.g., $- \frac{5}{8} + \frac{3}{8}$)

ENTRY POINTS and ACCESS SKILLS for The Number System Standards in Grade 6

← Less Complex

More Complex →

ACCESS SKILLS

The student will:

ENTRY POINTS

The student will:

Apply and extend previous understanding of rational numbers.
(continued)

- ◆ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted)
- ◆ Follow directions in counting sequence activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts)
- ◆ Respond to materials as they are counted in sequence
- ◆ In the context of an academic activity on addition (putting together and adding to) and subtraction (taking apart and taking from), respond to materials to be added or subtracted
- ◆ Track object as it is added or subtracted from set
- ◆ Attend visually, aurally, or tactilely to objects as they are counted

CONTENT AREA Mathematics
DOMAIN The Number System

Grade 7

Cluster	Standards as written	
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1a	Describe situations in which opposite quantities combine to make zero. <i>For example: A hydrogen atom has zero charge because its two constituents are oppositely charged; If you open a new bank account with a deposit of \$30 and then withdraw \$30, you are left with a \$0 balance.</i>
	7.NS.A.1b	Understand $p + q$ as the number located a distance $ q $ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide integers and other rational numbers.
	7.NS.A.2a	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
	7.NS.A.2b	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
	7.NS.A.2c	Apply properties of operations as strategies to multiply and divide rational numbers.
	7.NS.A.2d	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with integers and other rational numbers. ¹

¹ Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

ENTRY POINTS for The Number System Standards in Grade 7

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	<ul style="list-style-type: none"> ◆ Add positive and negative numbers using a number line, manipulatives or zero sums (e.g., represent $5 + (-3)$ on a number line by starting at 0, moving 5 units right, and then 3 units left) ◆ Add and subtract positive or negative multi-digit numbers and decimals (e.g., $61 - 24.5$, $77 - (-52)$, etc.) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add positive and negative numbers to solve a real-life problem (e.g., find the population of a town of 32022 people after 432 of them move away) ◆ Multiply and divide positive or negative multi-digit numbers or decimals (e.g., $16 \times (-10)$, $(-234) \div (-11.7)$, etc.) 	<ul style="list-style-type: none"> ◆ Add and subtract fractions (e.g., using any strategy) ◆ Multiply and divide fractions (e.g., $\frac{1}{3} \times \frac{3}{8}$, $\frac{2}{7} \div \frac{9}{5}$, etc.) ◆ Multiply and divide numbers to solve a real-life problem (e.g., find the weight of each tire if 12 of them weigh 252 pounds) ◆ Use any operation to compute with signed numbers (e.g., $-17 - (-20)$, $14 \times (-40)$, etc.) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN The Number System

Grade 8

Cluster	Standards as written	
Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion. For rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.
	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). <i>For example, by truncating the decimal expansion of $\sqrt{2}$ show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</i>

ENTRY POINTS for The Number System Standards in Grade 8

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Know that there are numbers that are not rational, and approximate them by rational numbers.	<ul style="list-style-type: none"> Order integers on a number line (e.g., plot integers on a number line given a point of reference) Compare decimals (e.g., given two numbers with decimals compare with symbols) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Order rational numbers on a number line (e.g., plot integers, decimals, and/or fractions on a number line given a point of reference) Compare decimals Convert fractions to decimal equivalent (e.g., convert $\frac{1}{8}$ to 0.125 and $\frac{2}{7}$ to 0.285714) Estimate square roots of rational numbers (e.g., $\sqrt{3} \approx 1.7$ and $\sqrt{22} \approx 4.7$) Classify numbers as rational or irrational (e.g., $\frac{13}{41}$, $2.\overline{33}$, 3.567, $\sqrt{25}$, and $\sqrt[3]{8}$ are rational, but $\sqrt{10}$ and 2π are not) 	<ul style="list-style-type: none"> Order rational and irrational numbers on a number line (e.g., given a number line and a point of reference) Solve problems involving square roots (e.g., if the area of a square is 42 square units, the length of each side is about 6.5 units) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: An <i>irrational number</i> is one that cannot be expressed as a quotient of two integers, e.g. $\sqrt{2}$. A number is irrational if, and only if, it cannot be written as a repeating or terminating decimal.</p> </div>

Ratios and Proportional Relationships

	Standards	Entry Points	Access Skills
6	Page 86	Pages 87 – 88	Pages 87 – 88
7	Page 89	Page 90	

CONTENT AREA Mathematics

DOMAIN Ratios and Proportional Relationships

Grade 6

Cluster	Standards as written	
Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.A.1	Understand the concept of a ratio including the distinctions between part: part and part: whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities. <i>For example: The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every two wings there was one beak; For every vote candidate A received, candidate C received nearly three votes, meaning that candidate C received three out of every four votes or $\frac{3}{4}$ of all votes.</i>
	6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ and use rate language in the context of a ratio relationship, <i>including the use of units</i> . <i>For example: This recipe has a ratio of three cups of flour to four cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar; We paid \$75 for 15 hamburgers, which is a rate of five dollars per hamburger.¹</i>
	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
	6.RP.A.3a	Make tables of equivalent ratios relating quantities with whole-number measurements. Find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
	6.RP.A.3b	Solve unit rate problems, including those involving unit pricing, and constant speed. <i>For example, if it took seven hours to mow four lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i>
	6.RP.A.3c	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.
	6.RP.A.3d	Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities. <i>For example, Malik is making a recipe, but he cannot find his measuring cups! He has, however, found a tablespoon. His cookbook says that 1 cup = 16 tablespoons. Explain how he could use the tablespoon to measure out the following ingredients: two cups of flour, $\frac{1}{2}$ cup sunflower seed, and $1\frac{1}{4}$ cup of oatmeal.²</i>
	6.RP.A.3e	Solve problems that relate the mass of an object to its volume.

¹ Expectations for unit rates in this grade are limited to non-complex fractions.

² Example is from the Illustrative Mathematics Project: <https://www.illustrativemathematics.org/content-standards/tasks/2174>

ENTRY POINTS and ACCESS SKILLS for Ratios and Proportional Relationships Standards in Grade 6

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Understand ratio concepts and use ratio reasoning to solve problems.	<ul style="list-style-type: none"> Respond to materials that demonstrate ratios and proportional relationships Track materials that demonstrate ratios and proportional relationships Shift focus from materials that demonstrate ratios and proportional relationships Grasp materials that demonstrate ratios and proportional relationships Use two hands to hold materials that demonstrate ratios and proportional relationships Release materials used to demonstrate ratios and proportional relationships Orient materials used to demonstrate ratios and proportional relationships Locate objects partially hidden or out of sight (e.g., remove barrier) to expose a ratio Use one object to act on another used to demonstrate ratios 	<ul style="list-style-type: none"> Create ratios among objects (e.g., ratio of circles to squares is 5:3) Identify a part-to-part relationship in a real-life situation using a proportion (e.g., 8 boys to 3 girls or 8:3 boys to girls) Express percentages using drawings or technology (e.g., $50\% = \frac{1}{2}$ of a circle) Express percentages as fraction equivalents (e.g., $75\% = \frac{75}{100}$) Convert simple measurement units (e.g., feet to yards or gallons to pints) Determine whether points graphed on a coordinate plane represent a proportional relationship (e.g., points which create a line that does not pass through the origin do not represent such a relationship) Identify equivalent fractions (e.g., given $\frac{1}{4}$, identify $\frac{2}{8}$ and $\frac{5}{20}$ as equivalents) 	<ul style="list-style-type: none"> Express part-to-part ratios in mathematical or real-life situations (e.g., 6 blue marbles to 8 green marbles represents a ratio of 6:8) Express the mathematical relationship of two related quantities as a ratio (e.g. in a bird the ratio of beaks to wings is 1:2 and the ratio of claws to beaks is 8:1) Plot equivalent ratios as ordered pairs, in the first quadrant of a coordinate plane (e.g. (2, 4), (3, 6), etc.) Convert measurement units from different measurement systems (e.g. kilometers to miles) Identify the factor used to obtain equivalent fractions (e.g., $\frac{1}{2} = \frac{3}{6}$ because $\frac{3}{3} = 1$ and $\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}$) Create equivalent fractions (e.g., given $\frac{3}{4}$ create an equivalent model of $\frac{6}{8}$) 	<ul style="list-style-type: none"> Express part-to-whole ratios in mathematical or real-life situations (e.g., given 6 blue marbles and 8 green marbles, $\frac{6}{14}$ of the marbles are blue) Calculate a unit rate between two given quantities (e.g., if 16 cupcakes are split between 8 children then unit rate is 2 cupcakes per child; 5 apples cost \$2.00 so the unit rate is \$0.40 per apple) Calculate unit rates in real-life problems to make comparisons (e.g., compare prices from Home Depot and Lowes by calculating unit rates) Convert rates by manipulating measurement units (e.g., miles per hour to feet per second) Match a fraction to its equivalents (e.g., given $\frac{1}{3}$ and a bank of fractions identify equivalents such as $\frac{2}{6}$ and/or $\frac{4}{12}$)

ENTRY POINTS and ACCESS SKILLS for Ratios and Proportional Relationships Standards in Grade 6

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Understand ratio concepts and use ratio reasoning to solve problems.	<ul style="list-style-type: none"> ◆ Turn on/off technology used to demonstrate ratios and proportional relationships (e.g., turn on voice-generating device to describe a relationship using “to/for every” language) ◆ Imitate action to create proportional relationships ◆ Initiate cause-and-effect response (e.g., turn on technology tool to activate ratio computer program) ◆ Sustain ratio and proportional relationship activity through response ◆ Gain attention during a ratio activity ◆ Make a request during ratio activity ◆ Choose materials to be distributed in a ratio and proportional relationship activity ◆ Attend visually, aurally, or tactilely to materials that demonstrate ratios and proportional relationships 	<ul style="list-style-type: none"> ◆ Calculate unit rates in real-life problems from a table showing a proportional relationship (e.g., determine the price of one item from a table showing various numbers and total costs of the item) ◆ Create equivalent fractions using numbers (e.g., given a fraction and another with a missing numerator or denominator name the unknown number) <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><i>Note: A proportional relationship is one in which the quotient of every y value to its corresponding x value is a constant. Graphically, at this level, the relationship is shown as a line in the first quadrant that passes through the origin.</i></p> </div>

CONTENT AREA Mathematics
DOMAIN Ratios and Proportional Relationships

Grade 7

Cluster	Standards as written	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. <i>For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.</i>
	7.RP.A.2	Recognize and represent proportional relationships between quantities.
	7.RP.A.2a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
	7.RP.A.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
	7.RP.A.2c	Represent proportional relationships by equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</i>
	7.RP.A.2d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
	7.RP.A.3	Use proportional relationships to solve multi-step ratio, rate, and percent problems. <i>For example, simple interest, tax, price increases and discounts, gratuities and commissions, fees, percent increase and decrease, percent error.</i>

ENTRY POINTS for Ratios and Proportional Relationships Standards in Grade 7

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Analyze proportional relationships and use them to solve real-world and mathematical problems.	<ul style="list-style-type: none"> ♦ Calculate a unit rate in the context of a described proportional relationship (e.g., a recipe that calls for 4 cups of flour and 2 cups of sugar has a ratio of 2 cups of flour to 1 cup of sugar) ♦ Find equivalent ratios for a given ratio in a real-life situation (e.g., if the ratio of girls to boys is 2:3, then for 40 girls there are 60 boys, and for 30 girls there are 45 boys) ♦ Calculate the percentage in a percent problem (e.g., what percent of 60 is 45?) ♦ Calculate the base in a percent problem (e.g., 9 is 75% of what number?) ♦ Calculate the unknown amount in a percent problem (e.g., what number is 50% of 8?) ♦ Solve one-step equations using multiplication (e.g., $3x=45$ or $4x=36$) ♦ Determine the amount of tax charged on an item given the item cost in whole dollars and the tax rate (e.g., the cost of an item is \$10, and the tax rate is 4%, the amount of tax charged is \$0.40) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ♦ Complete a table of a proportional relationship by filling in missing values (e.g., in a table, x values of 0, 3, __, 9, 18, and y values of 0, 2, 4, __, 12) ♦ Compute unit rates in terms of distance and time (e.g., If Jaycee traveled 20 miles in 30 minutes, then she traveled at a rate of 40 miles per hour) ♦ Solve percentage proportions where one missing quantity is represented by a variable (e.g., $8/40=x/100$) ♦ Create a proportion table given a ratio (e.g., given the ratio 1:3, create a table of equivalent ratios such as 2:6, 3:9, 6:18, 12:36, etc.) ♦ Identify the unit rate from a table, graph, equation, or a description (e.g., if 4 pounds of grapes cost \$9, determine is the cost of one pound) ♦ Determine whether values in a table represent a proportional relationship (e.g., do the values (3, 5), (6, 16), (15, 25) and (25, 75) in table form represent a proportion?) ♦ Determine the amount of tax charged on an item given the item cost and the tax rate (e.g., the cost of an item is \$18.99, and the tax rate is 5%, the amount of tax charged is \$0.95) ♦ Convert fractions to their percentage equivalent (e.g., convert $80/100$ to 80%, $\frac{1}{2}$ to 50%, and $\frac{3}{5}$ to 60%) ♦ Convert fractions to their decimal equivalent (e.g., convert $70/100$ to 0.7, $\frac{1}{8}$ to 0.125, and $15/40$ to 0.375) 	<ul style="list-style-type: none"> ♦ Solve proportions where one missing quantity is represented by a variable (e.g., $3/5=x/15$) ♦ Solve percentage decrease or increase problems (e.g., a dress that originally cost \$50 was on sale for 60% off; a train fare of \$14 was increased by 20%) ♦ Compute unit rates in a variety of contexts (e.g., miles per hour, cents per stick of gum, minutes per commercial) ♦ Solve equations that represent proportional relationships in real life (e.g., use $d = rt$ to find time if distance is 120 miles and the rate is 60 miles per hour) ♦ Solve mixed single-step percent problems using proportional relationships (e.g., what is 45% of 80?; 35 percent of what is 7?; 60 is what percent of 200?) ♦ Determine the total cost of an item given the item cost and the tax rate (e.g., the cost of an item is \$11.95 and the tax rate is 6%, the total cost of the item is \$12.67) ♦ Determine the tax rate given the pre-tax and the total costs (e.g., the cost of an item is \$599.00 and the total cost with tax is \$628.95, the tax rate is 5%) ♦ Solve word problems involving percentages that increase and/or decrease (e.g., sales price, mark up, sales tax)

Expressions and Equations

	Standards	Entry Points	Access Skills
6	Page 92 – 93	Pages 94 – 98, 100	Pages 94 – 101
7	Page 102	Pages 103 – 104	
8	Page 105 – 106	Pages 107 – 108	

CONTENT AREA Mathematics
DOMAIN Expressions and Equations

Grade 6

Cluster	Standards as written	
Apply and extend previous understandings of arithmetic to algebraic expressions.	6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.
	6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.
	6.EE.A.2a	Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation "Subtract y from 5" as $5 - y$.</i>
	6.EE.A.2b	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</i>
	6.EE.A.2c	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.</i>
	6.EE.A.3	Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.</i>
Reason about and solve one-variable equations and inequalities.	6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.</i>
	6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
	6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
	6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q , and x are all nonnegative rational numbers.

Reason about and solve one-variable equations and inequalities. (continued)	6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i>

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Apply and extend previous understandings of arithmetic to algebraic expressions.	<ul style="list-style-type: none"> ◆ Respond to materials to add, subtract, and/or count known and unknown quantities ◆ Track materials used to add, subtract, and/or count known and unknown quantities and create graphs ◆ Shift focus from materials used to add, subtract, and/or count known and unknown quantities, to speaker ◆ Grasp materials used to add, subtract, and/or count known and unknown quantities ◆ Use two hands to hold materials used to add, subtract, and/or count known and unknown quantities ◆ Release materials used to add, subtract, and/or count known and unknown quantities ◆ Move materials used to add, subtract, and/or count known and unknown quantities 	<ul style="list-style-type: none"> ◆ Represent repeated addition using equal groups of objects (e.g., arrange 12 objects into 3 groups of 4) ◆ Create groups of objects with equal amounts in multiple ways (e.g., given 12 objects, create 2 groups of 6, 3 groups of 4 etc.) ◆ Create equivalent expressions using the commutative property of addition (e.g., $5 + 3 = 3 + 5$) ◆ Create equivalent expressions using the commutative property of multiplication (e.g., $3 \times 4 = 4 \times 3$) ◆ Create equivalent expressions involving the distributive property (e.g. $3(2 + 7)$ can be written as $(3 \times 2) + (3 \times 7)$) ◆ Evaluate one-operation expressions (e.g., 5×2 or $7 - 4$) ◆ Represent addition from tables, charts, drawings, etc., as an expression (e.g., a group of 4 apples and a group of 3 apples can be represented by $4 + 3$) 	<ul style="list-style-type: none"> ◆ Use Order of Operations to evaluate a simple numerical expression involving multiplication and division (e.g., $30 \div 5 \times 3$) ◆ Use Order of Operations to evaluate a multi-step numerical expression involving addition and subtraction and multiplication (e.g., $25 - 3 \times 4 + 2$) ◆ Evaluate expressions with numbers and letters involving addition and subtraction, given the value of the unknown number (e.g., evaluate $6 - n$ for $n = 2, n = 3, \dots$) ◆ Evaluate expressions with numbers and letters involving multiplication given the value of the unknown number (e.g., evaluate $6x$ for $x = 4, x = 7, \dots$) ◆ Rewrite repeated multiplication using whole number exponents (e.g., 2×2 can be written as 2^2 or $5 \times 5 \times 5$ as 5^3) 	<ul style="list-style-type: none"> ◆ Use Order of Operations to evaluate a multi-step numerical expression involving addition or subtraction and multiplication or division (e.g., $16 - 4 \times 2$) ◆ Use Order of Operations to evaluate similar numerical expressions with parentheses inserted in different places (e.g., $4 + (5 \times 6) - 2$ and $4 + 5 \times (6 - 2)$) ◆ Evaluate expressions with numbers and letters involving multiple operations given the value of the unknown number (e.g., evaluate the expression $3n - 5$, if $n = 8, n = 9, \dots$) ◆ Rewrite repeated multiplication using whole number exponents and expressions with exponents as repeated multiplication (e.g., 7^3 can be written as $7 \times 7 \times 7$ and 8×8 can be written as 8^2)

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

← Less Complex

More Complex →

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	The student will:	The student will:	The student will:
Apply and extend previous understandings of arithmetic to algebraic expressions. (continued)	<ul style="list-style-type: none"> ♦ Orient materials used to add, subtract, and/or count known and unknown quantities (e.g., orient icon pictures used to label axis) ♦ Manipulate objects used to add, subtract, and/or count known and unknown quantities ♦ Locate objects partially hidden or out of sight needed to add, subtract, and/or count known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add, subtract, and/or count known and unknown quantities (e.g., use scissors to cut materials) ♦ Turn on technology used to add, subtract, and/or count known and unknown quantities (e.g., turn on technology tool to add and subtract) ♦ Imitate action used to add, subtract, and/or count known and unknown quantities (e.g., imitate classmate attaching icon to add) 	<ul style="list-style-type: none"> ♦ Represent multiplication with repeated addition using equal groups of objects or numbers (e.g., 5×3 is the same as $5 + 5 + 5$ or 3 groups of 5 objects) ♦ Create a multiplication expression from an array of objects or manipulatives (e.g., three rows of seven carrots can be written as 3×7) ♦ Represent multiplication from tables, charts, drawings, etc., as an expression (e.g., 15 apples in each of 3 baskets can be represented by 3×15) ♦ Demonstrate the commutative property of addition using manipulatives (e.g., 3 pencils + 2 pencils = 2 pencils + 3 pencils) ♦ Demonstrate the commutative property of multiplication using arrays (e.g., show that 3 arrays of 4 cubes has the same number of cubes as 4 arrays of three cubes) 	<ul style="list-style-type: none"> ♦ Rewrite numerical expressions with exponents as repeated multiplication (e.g., 7^4 can be written as $7 \times 7 \times 7 \times 7$) ♦ Evaluate numerical expressions involving addition and subtraction with parentheses (e.g., $(4 + 2) + (7 - 4)$) ♦ Simplify numerical expressions using the distributive property (e.g., $4(3 + 2) = (4 \times 3) + (4 \times 2) = 12 + 8 = 20$) ♦ Write an expression to represent a real-world situation using a variable for an unknown value (e.g., buying some sodas that cost \$1.50 each can be represented by $1.5n$) ♦ Create an expression involving addition or subtraction that represents a real-world situation (e.g., you have 16 tickets to sell and you sell t tickets; $16 - t$ tickets remain or adding 7 miles to a trip of m miles can be written as $m + 7$) 	<ul style="list-style-type: none"> ♦ Create expressions involving addition and subtraction with numbers and with a letter representing an unknown number in a real-life situation ♦ Create expressions involving multiplication and division that represents a real-world situation (e.g., 20 cookies shared by n students; each student gets $20 \div n$ cookies) ♦ Create expressions involving any operation with numbers and with a letter representing an unknown number in a real-life situation (e.g., a class that has 20 boys and n girls has $20 + n$ students) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
The student will:	The student will:	The student will:	The student will:
<p>Apply and extend previous understandings of arithmetic to algebraic expressions. (continued)</p> <ul style="list-style-type: none"> ◆ Initiate cause-and-effect response during an adding/subtracting/counting activity (e.g., turn on technology tool to activate addition computer program) ◆ Sustain adding/subtracting/counting activity through response (e.g., using preprogrammed voice-generating device comment) ◆ Gain attention during adding/subtracting/counting activity (e.g., raise hand, vocalize) ◆ Make a request during an activity to add, subtract, and/or count known and unknown quantities (e.g., request a turn) ◆ Choose from an array of two in an adding, subtracting, and/or counting activity (e.g., choose materials to be used in adding activity) ◆ Attend visually, aurally, or tactilely to materials to add, subtract, and/or count known and unknown quantities 	<ul style="list-style-type: none"> ◆ Identify equivalent numerical expressions (e.g., $8 + 8 + 8$ can be written as 3×8 or 8×3) ◆ Use Order of Operations to evaluate a simple numerical expression involving addition and subtraction (e.g., $10 - 3 + 4$) ◆ Describe a context in which a product is expressed (e.g., describe 3×12 as 3 dozen eggs) ◆ Describe a context in which a sum is expressed (e.g., describe $6 + 5$ as having six dollars and finding a five dollar bill) 	<ul style="list-style-type: none"> ◆ Create expressions involving multiplication with numbers and with a letter representing an unknown number in a real-life situation (e.g., Luke has n marbles and Audrey has 3 times as many, therefore Audrey has $3n$ marbles) 	

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Reason about and solve one-variable equations and inequalities.	<ul style="list-style-type: none">◆ Track materials used to add, subtract, and/or count known and unknown quantities◆ Shift focus from materials used to add, subtract, and/or count known and unknown quantities, to speaker◆ Grasp materials used to add, subtract, and/or count known and unknown quantities◆ Use two hands to hold materials used to add, subtract, and/or count known and unknown quantities◆ Release materials used to add, subtract, and/or count known and unknown quantities◆ Move materials used to add, subtract, and/or count known and unknown quantities◆ Respond to materials to add, subtract, and/or count known and unknown quantities and create graphs	<ul style="list-style-type: none">◆ Compare groups of objects to demonstrate the meaning of “equal to,” “greater than,” or “less than” (e.g., show that a set of 4 pens is “greater than” a set of 2 pens)◆ Answer yes/no questions about equations and inequalities◆ Compare number quantities using the symbols $<$, $=$, or $>$ (e.g., use the correct symbol when comparing 7 and 8 or 7 and 10)◆ Compare the magnitude of numbers on a number line using the symbols $<$ or $>$ (e.g., show that by their positions on the number line that 2 is less than 6)◆ Identify the missing number in an equation involving addition or subtraction (e.g., $_ + 8 = 13$)◆ Identify the missing number in an equation involving multiplication or division (e.g., $4 \times _ = 28$)	<ul style="list-style-type: none">◆ Solve addition and subtraction equations where the sum or difference is represented by a variable (e.g., $5 + 7 = r$ or $68 - 31 = m$)◆ Solve multiplication and division equations where the product or quotient is represented by a variable (e.g., $6 \times 7 = t$ or $18 \div 6 = 3$)◆ Solve simple inequalities using addition and subtraction when one of the quantities is unknown, using tables, technology, manipulatives, or drawings (e.g., solve $n - 4 > 11$ by showing that n must be greater than the number of objects subtracted plus the number of objects remaining)◆ Provide possible solutions of inequalities when one of the quantities is unknown (e.g., if $n > 4$, then n may be 5 or 8, but not 2)	<ul style="list-style-type: none">◆ Determine the range of solutions of inequalities involving multiplication and division when one quantity is unknown (e.g., if $2t > 6$, then t can be any number greater than 3 or $t > 3$)◆ Create equations involving any operation with numbers and with a letter representing an unknown number in real-life situations (e.g., A class that has 32 students has 20 boys and n girls can be written as $20 + n = 32$)◆ Determine the range of solutions of inequalities involving addition and subtraction when one quantity is unknown (e.g., if $4 + n > 9$, then n can be any number greater than 5 or $n > 5$)

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex

More Complex

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Reason about and solve one-variable equations and inequalities. (continued)	<ul style="list-style-type: none"> ♦ Orient materials used to add, subtract, and/or count known and unknown quantities (e.g., orient icon pictures used to label axis) ♦ Locate objects partially hidden or out of sight needed to add, subtract, and/or count known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add, subtract, and/or count known and unknown quantities (e.g., use scissors to cut materials) ♦ Turn on technology used to add, subtract, and/or count known and unknown quantities (e.g., turn on technology tool to create graph program) 	<ul style="list-style-type: none"> ♦ Distinguish between additive and multiplicative comparison when presented with a real-world situation (e.g., "five more than" vs. "five times as many") ♦ Solve equations using addition and subtraction when one of the quantities is unknown using tables, manipulatives, technology, or drawings (solve $10 - n = 4$ by showing how many pencils would need to be taken away from 10 pencils to leave 4 pencils) 	<ul style="list-style-type: none"> ♦ Assess the reasonableness of solutions of inequalities using mental computation (e.g., $19 + 16$ must be less than 40 since both addends are less than 20) ♦ Assess the reasonableness of answers to equations using mental computation (e.g., $19 + 21$ can't equal 100 since both addends are less than 50, but may equal 40 since both addends are close to 20) <p><i>Write an inequality to represent a real-life situation in one variable (e.g., I have 10 marbles and Tom has more marbles than I; represent Tom's marbles as $t > 10$)</i></p> <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ACCESS SKILLS (continued) for Expressions and Equations Standards in Grade 6

← Less Complex

More Complex →

ACCESS SKILLS The student will:

ENTRY POINTS The student will:

Reason about
and solve
one-variable
equations and
inequalities.
(continued)

- ◆ Imitate action used to add, subtract, and count known and unknown quantities (e.g., imitate classmate attaching icon to graph)
- ◆ Initiate cause-and-effect response during adding, subtracting, and/or counting activity (e.g., turn on technology tool to activate counting computer program)
- ◆ Sustain adding, subtracting, and/or counting activity through response (e.g., use preprogrammed voice-generating device comment)
- ◆ Gain attention during adding, subtracting, and/or counting activity (e.g., raise hand vocalize)
- ◆ Make a request during an activity to add, subtract, and/or count known and unknown quantities (e.g., request a turn)
- ◆ Choose from an array of two in an adding, subtracting, and/or counting activity (e.g., choose materials to be used in an adding activity)
- ◆ Attend visually, aurally, or tactilely to materials to add, subtract, and/or count known and unknown quantities

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Represent and analyze quantitative relationships between dependent and independent variables.	<ul style="list-style-type: none"> ◆ Respond to materials to create graphs ◆ Track materials used to create graphs ◆ Shift focus from materials used to create graphs, to speaker ◆ Grasp materials used to create graphs ◆ Use two hands to hold materials used to create graphs ◆ Release materials used to create graphs ◆ Move materials used to create graphs ◆ Orient materials used to create graphs (e.g., orient icon pictures used to label axis) ◆ Manipulate objects used to create graphs ◆ Locate objects partially hidden or out of sight needed to create graphs (e.g., remove barrier to expose materials) ◆ Use one object to act on another to create graphs (e.g., use scissors to cut materials) 	<ul style="list-style-type: none"> ◆ Represent unknown number quantities in an input-output table, given an input number and a rule (e.g., input is 60, rule is subtract 15) ◆ Answer yes/no questions about input/output tables ◆ Generate a number pattern given an initial value and an addition rule (e.g., initial value is 6, rule is "add 4", determine the next 5 numbers in the pattern) ◆ Generate a number pattern given an initial value and a subtraction rule (e.g., initial value is 50, rule is "subtract 7", determine the next 5 numbers in the pattern) 	<ul style="list-style-type: none"> ◆ Determine the addition rule in an input-output table (e.g., if input is 20 and output is 25, what is the rule?) ◆ Determine the subtraction rule in an input-output table (e.g., if input is 60 and output is 45, what is the rule?) ◆ Generate a number pattern given an initial value and a multiplication rule (e.g., initial value is 2, rule is "multiply by 3", find the next 4 numbers in the pattern) ◆ Extend a simple arithmetic sequence to illustrate a real-life situation (e.g., John earns \$20 a week for mowing the lawn) 	<ul style="list-style-type: none"> ◆ Create addition and subtraction equations to represent the relationship between two variables using graphs or tables (e.g., Input -15 = Output) ◆ Determine the multiplication rule in an input-output table (e.g., if input is 3 and output is 12, what is the rule?) ◆ Determine, given two variables, which is dependent, and which is independent in a given situation (e.g., the variables are "age" and "height", or "distance" and "time", which is the dependent variable?) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ACCESS SKILLS (continued) for Expressions and Equations Standards in Grade 6

Less Complex

More Complex

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Represent and analyze quantitative relationships between dependent and independent variables. (continued)	<ul style="list-style-type: none"> ◆ Turn on technology used to create graphs (e.g., turn on technology tool to create graph program) ◆ Imitate action used to create graphs (e.g., imitate classmate attaching icon to graph) ◆ Initiate cause-and-effect response during graphing activity (e.g., turn on technology tool to activate graphing computer program) ◆ Sustain graphing activity through response (e.g., use preprogrammed voice-generating device comment) ◆ Gain attention during graphing activity (e.g., raise hand vocalize) ◆ Make a request during an activity to create graphs (e.g., request a turn) ◆ Choose from an array of two during graphing activity (e.g., choose materials to be used in graphing activity) ◆ Attend visually, aurally, or tactilely to materials to create graphs 	

CONTENT AREA Mathematics
DOMAIN Expressions and Equations

Grade 7

Cluster	Standards as written	
Use properties of operations to generate equivalent expressions.	7.EE.A.1	Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. <i>For example, $4x + 2 = 2(2x + 1)$ and $-3(x - 5) = -3x + 15$.</i>
	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05." A shirt at a clothing store is on sale for 20% off the regular price, "p". The discount can be expressed as $0.2p$. The new price for the shirt can be expressed as $p - 0.2p$ or $0.8p$.</i>
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example, if a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; This estimate can be used as a check on the exact computation.</i>
	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4a	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i>
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. <i>For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</i>
	7.EE.B.4c	Extend analysis of patterns to include analyzing, extending, and determining an expression for simple arithmetic and geometric sequences (e.g., compounding, increasing area), using tables, graphs, words, and expressions.

ENTRY POINTS for Expressions and Equations Standards in Grade 7

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use properties of operations to generate equivalent expressions.	<ul style="list-style-type: none"> ◆ Demonstrate the commutative property of addition using manipulatives (e.g., show that $3 + 4 = 4 + 3$ by showing that regardless of their position there remains a total of 7 objects) ◆ Demonstrate the associative property of addition using manipulatives (e.g., show that $(2 + 1) + 7 = 2 + (1 + 7)$ by showing that regardless of the grouping there remains a total of 10 objects) ◆ Apply the commutative property of addition to expressions with positive and negative numbers (e.g., $12 + 9 = 9 + 12$ because both equal 21) ◆ Apply the associative property of addition to expressions with positive and negative numbers (e.g., $3 + (2 + 4) = (3 + 2) + 4$ because both equal 9) ◆ Match expressions with the property (commutative, associative, distributive) they exhibit (e.g., $(5 + 7) + 3 = 5 + (7 + 3)$ exhibits the associative property and $2(3 + 6) = (2 \times 3) + (2 \times 6)$ exhibits the distributive property) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Apply the distributive property to expressions with positive and negative numbers (e.g., $2(5 - 1) = (2 \times 5) - (2 \times 1)$ because $2 \times 4 = 10 - 2$) ◆ Apply the distributive property to simplify variable expressions (e.g., $3(t + 8) = 3t + 24$) ◆ Apply the commutative property of multiplication to expressions with positive and negative numbers (e.g., $8 \times 5 = 5 \times 8$ because both equal 40) ◆ Apply the associative property of multiplication to expressions with positive and negative numbers (e.g., $(6 \times 2) \times 5 = 6 \times (2 \times 5)$ because both equal 60) ◆ Apply the distributive property to evaluate expressions with positive and negative whole numbers (e.g., $6 \times 8 + 6 \times 2 = 6(8 + 2)$) 	<ul style="list-style-type: none"> ◆ Apply properties to simplify variable expressions involving one or more operations (e.g., $4(n - 3) + 7n = 4n - 12 + 7n = 11n - 12$ because of the distributive property and the commutative property of addition) ◆ Produce equivalent expressions by applying the distributive property to expressions with variables (e.g., $(10 \times a) + (10 \times b) = 10(a + b)$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Expressions and Equations Standards in Grade 7

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	<ul style="list-style-type: none"> ◆ Add and subtract simple fractions to solve a real-life problem using manipulatives or technology (e.g. combine fractions of a pizza using slices) ◆ Multiply and divide simple fractions to solve a real-life problem using manipulatives or technology (e.g., illustrate $\frac{2}{3} \times \frac{1}{2}$ by dividing $\frac{2}{3}$ cup of water into two equal portions and measuring the remaining amount in each cup) ◆ Multiply a two-digit number by a one-digit number to solve a real-life problem using manipulatives and drawings (e.g. using repeated addition) ◆ Divide a two-digit number by a one-digit number to solve a real-life problem using manipulatives and drawings (e.g. using repeated subtraction) ◆ Choose the number of objects within 20 that make a number sentence true to solve a real-life problem using manipulatives or drawings (e.g., compare groups of 5 and 11 objects and determine how many objects added to the five will yield 11) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add and subtract fractions or mixed numbers with the same denominator to solve a real-life problem (e.g., using quarters) ◆ Multiply a three-digit number by a one-digit number to solve a real-life problem (e.g., Shelby picks 110 tomatoes each day for a week) ◆ Divide a two-digit number by a one-digit number (with or without a remainder) to solve a real-life problem (e.g., Miranda will distribute 25 stickers to each of 5 friends) ◆ Add and subtract numbers, including decimals to tenths, to solve a real-life problem (e.g., Jamie rode her bike 1.6 miles to the playground and then 2.3 miles to the library; what is the total number of miles ridden?) ◆ Multiply and divide numbers, including decimals to tenths, to solve a real-life problem (e.g., Julio ran 4.6 miles and Lila ran 1.5 times as many miles; how many miles did Lila run?) 	<ul style="list-style-type: none"> ◆ Add and subtract fractions or mixed numbers to solve a real-life problem (e.g., Jackie ate $\frac{3}{4}$ sandwich for lunch and $1\frac{2}{3}$ sandwich for dinner; how much did she eat?) ◆ Multiply and divide fractions or mixed numbers to solve a real-life problem (e.g., find the total cost of $\frac{3}{4}$ lb. of coffee that costs $\\$5\frac{1}{2}$ per pound) ◆ Multiply a two- or three-digit number by a two-digit numbers to solve a real-life problem (e.g., Ramon saves \$35 per week for one year; what are his total savings?) ◆ Divide a three-digit number by a one-digit numbers (with or without a remainder) to solve a real-life problem (e.g., 105 M&Ms are shared among 7 students; how many for each student?) ◆ Add and subtract numbers, including decimals to hundredths, to solve a real-life word problem (e.g., Joy bought 2.325 pound of almonds and 3.7 pounds of pecans; what is the total number of pounds of nuts Joy bought?) ◆ Use any operation to solve a real-life problem (e.g., Include addition/subtraction and multiplication/division problems) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Expressions and Equations

Grade 8

Cluster	Standards as written	
Work with radicals and integer exponents.	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $32 \div 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</i>
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
	8.EE.A.3	Use numbers expressed in the form of a single digit multiplied by an integer power of 10 to estimate very large or very small quantities, and express how many times as much one is than the other. <i>For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.</i>
	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Understand the connections between proportional relationships, lines, and linear equations.	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i>
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.C.7	Solve linear equations in one variable.
	8.EE.C.7a	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
	8.EE.C.7b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations.

	8.EE.C.8a	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
	8.EE.C.8b	Solve systems of two linear equations in two variables algebraically (using substitution and elimination strategies), and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</i>
	8.EE.C.8c	Solve real-world and mathematical problems leading to two linear equations in two variables. <i>For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</i>

ENTRY POINTS for Expressions and Equations Standards in Grade 8

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Work with radicals and integer exponents.	<ul style="list-style-type: none"> Express the notion of a mathematical square given a model of a square using both a multiplication expression and an exponent (e.g., given a square, express its dimensions as both 7×7 and 7^2) Evaluate integers raised to the second power (e.g., 2^2, 8^2 etc.) Rewrite numbers greater than 10 using scientific notation (e.g., rewrite 25.4 as 2.54×10) Identify perfect squares within 100 (e.g., 64 is a perfect square, but 60 is not) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Rewrite repeated multiplication using exponents (e.g., rewrite $2 \times 2 \times 2$ as 2^3) Rewrite numbers with exponents as repeated multiplication (e.g., rewrite 8^4 as $8 \times 8 \times 8 \times 8$) Evaluate integers raised to any positive power (e.g., 4^3, 8^5 etc.) Rewrite numbers greater than 100 using scientific notation (e.g., rewrite 172.2 as 1.722×10^2) Evaluate square roots when possible (e.g., $\sqrt{25}$ is 5, but $\sqrt{23}$ can't be simplified) Identify perfect squares within 1000 (e.g., 121 is a perfect square, but 125 is not) Identify perfect cubes within 1000 (e.g., 216 is a perfect cube, but 225 is not) 	<ul style="list-style-type: none"> Rewrite numbers using scientific notation (e.g., rewrite 0.0556 as 5.56×10^{-2}) Rewrite numbers shown in scientific notation in standard notation (e.g., rewrite as 9.07×10^3 as 9070) Evaluate square roots or cube roots when possible (e.g., $\sqrt[3]{27}$ is 3, but $\sqrt[3]{36}$ can't be simplified) Translate between standard notation and scientific notation (e.g., 44180 is 4.418×10^4 and 6×10^{-1} is 0.6) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Understand the connections between proportional relationships, lines, and linear equations.	<ul style="list-style-type: none"> Use proportions to solve real-life problems using unit rate (e.g., four apples cost 80 cents, so one apple costs 20 cents) Identify a specific data point when given the coordinates (e.g., locate (3,8) on a coordinate grid) Use proportions to solve real-life problems (e.g., in Jasmine's homeroom, the ratio of girls to boys is 3:2. If there are 12 girls in Jasmine's homeroom, how many boys are there?) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Calculate a unit rate from a description of a relationship (e.g., if 2 jars of jam cost \$4.00, how much does 1 jar cost?) Plot points from a proportional relationship in the first quadrant of a coordinate plane (e.g., plot points listed in a table) Graph a simple proportional relationship when given the unit rate (if the unit rate is 5:1, graph the line through the origin and (1,5)) Given a graph of a proportional relationship, determine the unit rate (e.g., determine the slope of a line through the origin and (2,3)) 	<ul style="list-style-type: none"> Create graphs on a coordinate plane to represent a proportional relationship (e.g., from values in a table) Compare two or more lines on a coordinate grid to determine which has the greater unit rate (i.e., slope) Create a graph of a proportional relationship given an equation (e.g., graph $y = \frac{3}{2}x$) Given a graph of a proportional relationship, write the equation that represents the relationship <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Expressions and Equations Standards in Grade 8

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Analyze and solve linear equations and pairs of simultaneous linear equations.	<ul style="list-style-type: none"> ◆ Demonstrate the relationship between addition and subtraction as inverse operations (one undoes the other) using manipulatives or drawings ◆ Demonstrate the relationship between multiplication and division as inverse operations (one undoes the other) using manipulatives or drawings ◆ Solve a one-step equation involving addition and/or subtraction (e.g., $x - 7 = 18$) ◆ Solve a one-step equation involving multiplication and/or division (e.g., $14 \div n = 7$) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify the operation needed to solve an algebraic equation (e.g., to solve $n + 5 = 12$, one must subtract 5 or add -5) ◆ Solve one or two-step equations (e.g., $2x - 7 = 15$) ◆ Graph a two-variable equation on a coordinate grid. (e.g., graph the equation $y = \frac{1}{2}x - 3$) ◆ Identify the coordinates of the point where two lines on a coordinate grid intersect (not including grid lines) 	<ul style="list-style-type: none"> ◆ Determine whether a system of equations has one solution or no solutions from a graph (e.g., if the lines are parallel the system has no solution) ◆ Solve a system of linear equations in two variables algebraically (e.g., given the equations $y = 2x$ and $y = -3x + 5$ solve for x by setting $2x$ equal to $-3x + 5$, and then using substitution) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

Functions

	Standards	Entry Points	Access Skills
8	Page 110	Pages 111 – 113	Pages 111 – 114

CONTENT AREA Mathematics
DOMAIN Functions

Grade 8

Cluster	Standards as written	
Define, evaluate, and compare functions.	8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. ¹
	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i>
	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. <i>For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.</i>
Use functions to model relationships between quantities.	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

¹ Function notation is not required in grade 8.

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex

More Complex

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Define, evaluate, and compare functions. <ul style="list-style-type: none"> Respond to materials being compared by quantity or size Track materials being compared by quantity or size Shift focus from materials being compared by quantity or size to speaker Grasp materials being compared by quantity or size Release materials being compared by quantity or size Give materials being compared by quantity or size Move materials being compared by quantity or size Orient materials being compared by quantity or size Construct two objects using materials from two sets of materials being compared by quantity or size (e.g., build two block towers, with one set of 3 blocks and one with 5 blocks) Turn on voice-generating device to participate in an activity to compare materials by quantity or size 	<ul style="list-style-type: none"> Answer yes/no questions about functions Identify the input (x) and the output (y) in an input-output table and interpret the relationship between one variable and another variable in a table (e.g., number of hours, number of miles traveled) Create an input-output table when given the input values and the function rule Graph a function on the coordinate plane using a completed table Represent unknown number quantities in an input-output table, given an input number and a rule (e.g., input = 60, rule is subtract 15, then output = 45) Answer yes/no questions about input/output tables Generate a number pattern given an addition rule and an initial value (e.g., start with 6, rule is add 4, find the next 5 numbers in the pattern) 	<ul style="list-style-type: none"> Identify linear and non-linear functions from given tables by graphing them Explain why the relationships are or are not functions in given tables (e.g., given a table with x-input: 9, 9, 16, 16 and corresponding y-output: 3, -3, 4, -4 determine the relationship is not a function because some inputs have more than one output) Explain why the relationships are or are not functions in given graphs (e.g., a graph of a circle is not a function because for some x-inputs there is more than one y-output; a horizontal line on the coordinate plane is a function because for every x input there is only one y output) Determine the addition rule in an input-output table (e.g., if input is 20 and output is 25, what is the rule?) 	<ul style="list-style-type: none"> Explain why the relationships are or are not functions in given algebraic equations (e.g., the equation $y=x$ is a function because each input has only one output and $y^2 + x^2 = 25$ is not a function because one input has 2 outputs) Compare <i>slopes</i> of two functions presented in different ways (e.g., the algebraic representation of the function $y = 3x + 2$ has a positive slope and the table representation of the function X input 0, 1, 2 and corresponding y outputs 2, 0, -2 has a negative slope) Determine whether a function is linear or non-linear by graphing it Describe how varying the rate of change in a variable affects the outcome in a table (e.g., increasing the speed decreases the time needed to arrive at a destination)

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Define, evaluate, and compare functions. (continued)	<ul style="list-style-type: none"> ♦ Imitate action with materials being used to compare quantity or size ♦ Sustain activity comparing objects by size or quantity through response ♦ Gain attention in activity comparing objects by size and quantity (e.g., raise hand during comparison lesson on white board) ♦ Imitate action using materials compared by quantity or size ♦ Initiate cause-and-effect using materials compared by quantity or size ♦ Make a request in an activity comparing materials by size or quantity ♦ Choose from an array of two in an activity to compare materials by quantity ♦ Choose beyond an array of two to compare materials by quantity ♦ Attend visually aurally, or tactilely to materials being compared by quantity or size 	<ul style="list-style-type: none"> ♦ Generate a number pattern given a subtraction rule and an initial value (e.g., start with 50, rule is subtract 7, find the next 5 numbers in the pattern) ♦ Compare <i>initial values</i> of two functions presented in different ways (e.g.: Compare the initial value in Sam and Tom's savings account in the description and equations below: <u>Description for Sam:</u> Sam deposits \$20 in the bank to open an account. He puts \$10 into the account each month and withdraws nothing. <u>Equation for Tom:</u> $s = 4m + 200$ Answer: Sam's initial value is \$20, and Tom's is \$200. Tom has a greater initial value) ♦ Classify graphs of functions as linear or nonlinear given two graphical representations 	<ul style="list-style-type: none"> ♦ Determine the subtraction rule in an input-output table (e.g., if input is 60 and output is 45, what is the rule?) ♦ Generate a number pattern given a multiplication rule and an initial value (e.g., start with 2, rule is multiply by 3, find the next 4 numbers in the pattern) ♦ Compare rates of change of two functions presented in different ways (e.g.: Compare the rate of change in Sam and Tom's savings in the description and equations below. Description for Sam: Sam deposits \$20 in the bank to open an account. He puts \$10 into the account each month and withdraws nothing. Equation for Tom: $s = 4m + 200$ Answer: Sam puts \$10 in his account every month and Tom puts \$4 in his account every month, so Sam's saving has a greater rate of change)
			Express addition and subtraction equations to represent the relationship between two variables using graphs or tables (e.g., Input -15 = Output) Determine the multiplication rule in an input-output table (e.g., if input is 3 and output is 12, what is the rule?) <i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i>

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex

More Complex

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>		<u>The student will:</u>	<u>The student will:</u>
Use functions to model relationships between quantities.	<ul style="list-style-type: none"> Respond to materials to create graphs Track materials used to create graphs Shift focus from materials used to create graphs to speaker Grasp materials used to create graphs Use two hands to hold materials used to create graphs Release materials used to create graphs Move materials used to create graphs Orient materials used to create graphs (e.g., orient icon pictures used to label axis) Manipulate objects used to create graphs Locate objects partially hidden or out of sight needed to create a graph (e.g., remove barrier to expose materials) Use one object to act on another to create graphs (e.g., use scissors to cut materials) 	<ul style="list-style-type: none"> Determine the initial value (y-intercept) and the rate of change (slope) from graphs Match given descriptions to a given graph (e.g., match segments on a graph to segments of the following description: Jenny is walking to Samantha's house at a constant rate. Jenny gets to Samantha's house and is waiting. Jenny and Samantha ride the bus to school. The bus is moving at a constant rate but much faster than Jenny's walking rate.) Answer yes/no questions about graphs 	<ul style="list-style-type: none"> Determine the initial value (y-intercept) from graphs and linear equations Determine the rate of change (slope) from graphs and linear equations Describe a graph of a function that has labeled sections (e.g., Describe that Part A rises gently, Part B is short but flat, Part C rises like part A, but is steeper, etc.) Determine the initial value (y-intercept) from tables, graphs, linear equations, and descriptions Determine the rate of change (slope) from tables, graphs, linear equations and descriptions Sketch a graph from given context (e.g., given the context "Jenny is walking to Samantha's house at a constant rate" the student draws a straight line on the graph with a slope. And labels time on the x-axis and distance on the y-axis. When the context changes, "Jenny gets to Samantha's house and is waiting" the student draws a horizontal line, etc.)

Continue to address skills and concepts that approach grade-level expectations in this cluster

ACCESS SKILLS (continued) for Functions Standards in Grade 8

Less Complex

More Complex

<u>ACCESS SKILLS</u> <u>The student will:</u>		<u>ENTRY POINTS</u> <u>The student will:</u>
Use functions to model relationships between quantities. (continued)	<ul style="list-style-type: none"> ◆ Turn on technology used to create graphs ◆ Imitate action used to create graphs (e.g., imitate classmate attaching icon to graph) ◆ Initiate cause-and-effect response to turn on technology tool to activate graphing computer program ◆ Sustain graphing activity through response (e.g., using preprogrammed voice-generating device comment) ◆ Gain attention in a graphing activity (e.g., raise hand vocalize) ◆ Make a request during a graphing activity (e.g., request a turn) ◆ Choose from materials to be used in graphing activity ◆ Attend visually, aurally, or tactilely to materials to create graphs 	

Measurement and Data

	Standards	Entry Points	Access Skills
PK	Page 116	Pages 117, 119, 121	Pages 117 – 121
K	Page 122	Page 123	
1	Page 124	Pages 125 – 126	Pages 125 – 126
2	Page 127	Pages 128 – 129	
3	Pages 130 – 131	Pages 132 – 133	
4	Pages 134 – 135	Page 136	
5	Page 137	Page 138	

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Pre-Kindergarten		
Cluster	Standards as written	
Describe and compare measurable attributes.	PK.MD.A.1	Recognize the attributes of length, area, weight, and capacity of everyday objects using appropriate vocabulary (e.g., long, short, tall, heavy, light, big, small, wide, narrow).
	PK.MD.A.2	Compare the attributes of length and weight for two objects, including longer/shorter, same length; heavier/lighter, same weight; holds more/less, holds the same amount.
Classify objects and count the number of objects in each category.	PK.MD.B.3	Sort, categorize, and classify objects by more than one attribute.
Work with money.	PK.MD.C.4	Recognize that certain objects are coins and that dollars and coins represent money.

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Pre-K

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Describe and compare measurable attributes.	<ul style="list-style-type: none"> Respond to sets of materials presented by measurable attributes (e.g., big/small, long/short, heavier/lighter, less/more) Track materials that are labeled by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) Shift gaze from speaker to materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) Grasp materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) 	<ul style="list-style-type: none"> Answer yes/no questions about subject matter about comparing measurable attributes of objects 	<ul style="list-style-type: none"> Identify an object by one measurable attribute (e.g., show me the big ball) Compare two objects based on length or width

Attributes must be measurable, such as tall vs short, but NOT colors or textures

ACCESS SKILLS (continued) for Measurement and Data Standards in Pre-K

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>
<u>The student will:</u>		<u>The student will:</u>
Describe and compare measurable attributes. (continued)	<ul style="list-style-type: none"> ◆ Release materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ◆ Give materials described by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ◆ Move materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ◆ Manipulate objects with measurable attributes with two hands ◆ Locate partially hidden objects remove barrier between objects to compare measurable attributes ◆ Use one object to act on another ◆ Adjust planes to move objects with measurable attributes ◆ Make a request with the materials to be compared by measurable attribute ◆ Choose from an array of two to compare by measurable attribute with the presented object ◆ Attend (visually, aurally, or tactilely) to sets of materials labeled "same" or "different" 	

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Pre-K

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>		<u>The student will:</u>	<u>The student will:</u>
Classify objects and count the number of objects in each category.	<ul style="list-style-type: none"> ◆ Respond to materials as they are counted ◆ Shift focus from materials to speaker counting materials ◆ Grasp materials as they are counted ◆ Release materials as they are counted ◆ Give materials as they are counted ◆ Move objects as they are counted ◆ Orient objects as they are counted (e.g., turn flowerpots upright) ◆ Manipulate objects with two hands as they are counted ◆ Locate objects partially hidden or out of sight, to add or subtract to a collection of objects to be counted ◆ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ◆ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall) 	<ul style="list-style-type: none"> ◆ Collect objects with similar characteristics (e.g., all round objects) 	<ul style="list-style-type: none"> ◆ Sort items by one given measurable attribute (e.g., sort the pencils by length) ◆ Classify sorted objects based on measurable attribute (e.g., long and short pencils)

ACCESS SKILLS (continued) for Measurement and Data Standards in Pre-K

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>
<u>The student will:</u>		<u>The student will:</u>
Classify objects and count the number of objects in each category. (continue)	<ul style="list-style-type: none"> ♦ Construct using materials that have been counted in sequence (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence activity ♦ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Gain attention in counting sequence activity ♦ Make a request in counting sequence ♦ Sustain counting sequence activity through response activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Follow directions in counting sequence activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts) ♦ Respond to materials as they are counted in sequence ♦ Attend visually, aurally, or tactilely to materials as they are counted 	

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Pre-K

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>
	<u>The student will:</u>	<u>The student will:</u>
Work with money.	<ul style="list-style-type: none"> ◆ In the context of an academic activity, work with money ◆ Respond to bills and coins ◆ Track money ◆ Shift focus from money to speaker ◆ Grasp bills or coins ◆ Release bills or coins ◆ Give bills or coins ◆ Locate money partially hidden or out of sight (e.g., remove barrier to expose coins or bills) ◆ Adjust planes to move money ◆ Turn on/off (e.g., turn on prerecorded message about money on a voice-generating device) ◆ Imitate action (e.g., imitate a cash exchange) ◆ Initiate cause-and-effect response (e.g., initiate a cause-and-effect money computer program) ◆ Sustain a money activity through response ◆ Gain attention in an academic activity related to money ◆ Make a request in a money lesson (e.g., request a turn) ◆ Attend visually, aurally, or tactilely to bills and coins 	<ul style="list-style-type: none"> ◆ Answer yes/no questions related to money ◆ Select a coin from other objects as something of value

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Kindergarten		
Cluster	Standards as written	
Describe and compare measurable attributes.	K.MD.A.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object
	K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. <i>For example, directly compare the heights of two children and describe one child as taller/shorter.</i>
Classify objects and count the number of objects in each category.	K.MD.B.3	Classify objects into given categories; count the numbers of objects in each category (up to and including 10) and sort the categories by count.

ENTRY POINTS for Measurement and Data Standards in Kindergarten

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Describe and compare measurable attributes.		♦ Use measurable attributes to describe objects (e.g., the book is heavy, that shoe is long)	♦ Given two objects identify the object that is bigger or smaller.
Classify objects and count the number of objects in each category.	♦ Count the number of presorted objects	♦ Count the number of objects with similar characteristics (e.g., given a mixed group of shapes, count the number of squares)	♦ Group each pre-sorted set by the number of objects in each set (e.g., both blue and green sets of buttons have 4)

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 1

Cluster	Standards as written	
Measure lengths indirectly and by iterating length units.	1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
	1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
Tell and write time.	1.MD.B.3	Tell and write time in hours and half-hours using analog and digital clocks.
Represent and interpret data.	1.MD.C.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Work with money.	1.MD.D.5	Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to five pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems (up to 100¢).

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Grade 1

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Measure lengths indirectly and by iterating length units.	<ul style="list-style-type: none"> ◆ Order objects by length ◆ Use measurable attributes to describe 3 objects different in length. (e.g., the marker is longer than the crayon but shorter than the pencil) ◆ Answer questions related to length and/or height with multiple objects (e.g., which is longer?) ◆ Identify from a group of similar objects which one is longest, tallest, widest, or deepest 	<ul style="list-style-type: none"> ◆ Tell how many units long an object is that has been premeasured with multiple copies of the shorter object (the length unit) end to end (e.g., this pencil is 6 paper clips long) 	<ul style="list-style-type: none"> ◆ Express the length of an object as a whole number of length units by laying multiple copies of the shorter object (the length unit) end to end (e.g., lay rectangular blocks end to end <i>without</i> overlaps or spaces until they reach the same length, for example, as the side of a bookshelf. Students should be given objects that match up exactly)
Tell and write time.	<ul style="list-style-type: none"> ◆ Respond to daily schedule ◆ Track daily schedule materials as they are presented ◆ Shift gaze from speaker to daily schedule ◆ Grasp daily schedule pictures or icons ◆ Hold daily schedule pictures or icons ◆ Release daily schedule pictures or icons ◆ Give daily schedule pictures or icons ◆ Orient daily schedule pictures or icons 	<ul style="list-style-type: none"> ◆ Using daily schedule or other sequence of events, determine "now, later" or "first, then" ◆ Identify intervals of time (e.g., morning, afternoon, evening) ◆ Demonstrate understanding of the concepts today/tomorrow/yesterday ◆ Tell time to the nearest hour and/or half hour using digital clock 	<ul style="list-style-type: none"> ◆ Sequence events using representation of daily schedule ◆ Tell time to 5 minute intervals on an analog clock ◆ Tell time to the hour and/or half-hour on an analog clock ◆ Tell time to the nearest hour and/or half hour using analog clock (e.g. given 6:38 on an analog clock what is the nearest hour)

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Grade 1

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>
	<u>The student will:</u>	<u>The student will:</u>
Tell and write time. (continued)	<ul style="list-style-type: none"> ◆ Locate daily schedule pictures or icons partially hidden or out of sight ◆ Turn on/off time-monitoring device ◆ Gain attention to request a turn in an activity related to clocks ◆ Make a request in an activity in telling time ◆ Choose from an array of two choose materials in a clock or time activity ◆ Attend (visually, aurally, or tactilely) to daily schedule 	
Represent and interpret data.		<ul style="list-style-type: none"> ◆ Organize objects or symbols collected by two characteristics on a template (e.g., girls' and boys' favorite ice cream flavor) ◆ Represent data using an organizational tool (e.g., chart, list, tally, table) ◆ Answer questions to interpret data that has been organized into categories (e.g., how many liked chocolate ice cream?)
Work with money.		<ul style="list-style-type: none"> ◆ Identify U.S. coins ◆ Use appropriate notation to express the value of a coin or a combination of coins ◆ Identify coins needed for common purchases ◆ Identify the value of U.S. coins ◆ Find the sum of the values of a mixed group of coins ◆ Identify pennies by value ◆ Identify two U.S. coins by name ◆ Group and compare familiar objects (e.g., coins, quantities) ◆ Determine equivalent values of U.S. coins (e.g., five pennies equal one nickel) ◆ Solve simple problems involving the values of coins up to \$0.99 ◆ Identify U.S. coins by name and value ◆ Identify nickels and dimes by value

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 2

Cluster	Standards as written	
Measure and estimate lengths in standard units.	2.MD.A.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
	2.MD.A.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
	2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.
	2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
Relate addition and subtraction to length.	2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
	2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
Work with time and money.	2.MD.C.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
	2.MD.C.7a	Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week; days in a month and a year and approximate number of weeks in a month and weeks in a year.
	2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies (up to \$10), using \$ and ¢ symbols appropriately and whole dollar amounts. <i>For example, if you have 2 dimes and 3 pennies, how many cents do you have? If you have \$3 and 4 quarters, how many dollars or cents do you have? (Students are not expected to use decimal notation.)</i>
Represent and interpret data.	2.MD.D.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record the data on a line plot (dot plot) where the horizontal scale is marked off in whole-number units.
	2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems, using information presented in a bar graph.

ENTRY POINTS for Measurement and Data Standards in Grade 2

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Measure and estimate lengths in standard units.	<ul style="list-style-type: none"> Identify the correct tool to use given a situation requiring measurement <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Measure and compare the length of objects using a non-standard measuring device 	<ul style="list-style-type: none"> Measure length of objects using pre-selected standard tools (e.g., rulers, yardsticks, and meter sticks) Record and compare the measured length of objects using 2 pre-selected standard tools (rulers, yardsticks, and meter sticks) by repeating the use of the measurement tool/unit without gaps or overlaps (e.g., it takes a smaller number of feet to measure the height of the door than it does inches) Identify which one is longest, tallest, widest, or deepest from a group of similar objects using standard units of measurement Use two <i>different</i> standard units of measurement to describe an object (e.g., inches, feet, or yards) Estimate how many units of measurement are needed for a certain object <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Relate addition and subtraction to length.	<p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Solve addition and subtraction word problems within 20 involving lengths that are given in the same units using a visual model Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers (0, 1, 2.....) Represent whole number sums and differences within 20 on a number line 	<ul style="list-style-type: none"> Solve addition and subtraction word problems within 50 involving lengths that are given in the same units using a visual model Represent sums and differences of whole number lengths within 50 on a number line <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Measurement and Data Standards in Grade 2

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Work with time and money.	<ul style="list-style-type: none"> Identify \$1, \$5, and \$10-dollar bills <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Tell time on analog to the half hour Tell time on analog clock to the half hour, including a.m. and p.m. Identify the value of a mixed array of bills and coins Express the value of money as dollars and cents Express time to the nearest five minutes on an analog clock 	<ul style="list-style-type: none"> Tell time on analog clock to the quarter hour. Tell time on analog clock to the quarter hour, including a.m. and p.m. Solve simple problems involving the values of coins up to 99 cents <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Represent and interpret data.	<ul style="list-style-type: none"> Record measurement data for multiple objects using a single unit scale Create a graph to represent a data set <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Answer questions using measurement data represented on a graph (e.g., plant heights) 	<ul style="list-style-type: none"> Generate length measurement data to the nearest inch using a ruler Construct a line plot using measurement data with multiple data points using whole numbers Answer questions using measurement data represented on a line plot Solve word problems using information presented on a bar or picture graph <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 3

Cluster	Standards as written	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. ¹
Represent and interpret data.	3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent five pets.</i>
	3.MD.B.4	Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot (dot plot), where the horizontal scale is marked off in appropriate units—whole numbers, halves, or fourths.
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement.
	3.MD.C.5a	A square with side length of one unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
	3.MD.C.5b	A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
	3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in., square ft., and non-standard units).
	3.MD.C.7	Relate area to the operations of multiplication and addition.
	3.MD.C.7a	Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
	3.MD.C.7b	Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
	3.MD.C.7c	Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the Distributive property in mathematical reasoning.

¹ Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (continued)	3.MD.C.7d	Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	3.MD.D.8	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

ENTRY POINTS for Measurement and Data Standards in Grade 3

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	<ul style="list-style-type: none"> ◆ Determine intervals of time before and/or after a given time (e.g., what time is 1 hour before 3:05?) ◆ Choose the most appropriate measurement unit for an associated object (e.g., use grams to describe the weight of an apple) ◆ Choose the most appropriate tool to measure an associated object (e.g., use a tape measure for a pumpkin) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Use liters to express liquid volume measurements ◆ Use scales to measure mass in grams and kilograms ◆ Solve simple word problems involving addition and subtraction of time intervals of one or more hours 	<ul style="list-style-type: none"> ◆ Express time to the nearest minute on an analog clock ◆ Make estimates of liquid volume without measuring (e.g., is there enough water in the bucket to fill four liters?) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Represent and interpret data.	<ul style="list-style-type: none"> ◆ Draw a scaled picture graph (e.g., each smiley face represents 5 kids) ◆ Answer questions about "more" and "less" using line plot data <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Answer questions based on a data display (e.g., bar graph, picture graph) ◆ Answer questions using data represented on a graph 	<ul style="list-style-type: none"> ◆ Answer questions based on a scaled picture and/or scaled bar graph ◆ Generate length measurement data to the nearest half-inch using a ruler (measure and record) ◆ Construct a line plot with multiple measurement data points using a half-inch scale ◆ Answer questions based on line plot data ◆ Create a scaled bar graph (e.g., each square represents 5 pets) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Measurement and Data Standards in Grade 3

Less Complex

More Complex

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	<ul style="list-style-type: none"> Find the area of regular and/or irregular shapes by tiling an area and counting unit squares using manipulatives, technology, or visual models <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Find the area of rectangular by repeated addition of either columns or rows of tiles that comprise a rectangle (e.g., add tiles in a 3 by 4 rectangle to show that $3 + 3 + 3 + 3 = 12$ square units or $4 + 4 + 4 = 12$ square units) 	<ul style="list-style-type: none"> Find the area of rectangle by multiplying side lengths <p>Continue to address skills and concepts that approach grade-level expectations in this cluster</p>
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	<p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Measure the sides of a polygon to the nearest whole unit 	<ul style="list-style-type: none"> Calculate the perimeter of polygons <p>Continue to address skills and concepts that approach grade-level expectations in this cluster</p>

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 4

Cluster	Standards as written	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	4.MD.A.1	Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i>
	4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
	4.MD.A.3	Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (Note: When finding areas of rectangular regions answers will be in square units. For example, the area of a 1 cm x 1 cm rectangular region will be 1 square centimeter (1 cm², students are not expected to use this notation.) When finding the perimeter of a rectangular region answers will be in linear units. For example, the perimeter of the region is: 1cm + 1cm + 1cm + 1cm = 4 cm or 2(1cm) + 2(1cm) = 4 cm)</i>
Represent and interpret data.	4.MD.B.4	Make a line plot (dot plot) representation to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots (dot plots). <i>For example, from a line plot (dot plot) find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i>
Geometric measurement: Understand concepts of angle and measure angles.	4.MD.C.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: (5a, 5b)
	4.MD.C.5a	An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles
	4.MD.C.5b	An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
	4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

	4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
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ENTRY POINTS for Measurement and Data Standards in Grade 4

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	<ul style="list-style-type: none"> Compare relative sizes of standard units of measurement by using manipulatives and record findings (e.g., record how to determine that a yard is equivalent to 3 feet by laying 3 rulers end-to-end next to a yard stick) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Measure and record the linear measurements of an object using two standard units of measure (e.g., measure a board first in centimeters and then millimeters) Measure and record liquid measurements using two standard units of measure (e.g., quarts and gallons) Measure and record time measurements using two standard units of measurement (e.g., hours and minutes) 	<ul style="list-style-type: none"> Solve word problems involving addition or subtraction for distances using a number line Solve word problems involving addition or subtraction for intervals of time using a number line Solve word problems involving addition or subtraction for liquid volume using a number line Solve word problems involving addition or subtraction for masses of objects using a number line Solve word problems involving addition or subtraction for money using a number line Solve area problems using visual models Solve perimeter problems using visual models <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Represent and interpret data.	<p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Generate length measurement data to the nearest $\frac{1}{4}$-inch using a ruler Construct a line plot with multiple data points using fractions 	<p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Geometric measurement: Understand concepts of angle and measure angles.	<ul style="list-style-type: none"> Measure angles using labeled circle wedges (e.g., 90-degree, 45 degree or 120 degree wedges) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate</i></p>	<ul style="list-style-type: none"> Measure in whole number degrees angles using a protractor 	<ul style="list-style-type: none"> Sketch angles of a specified measure Solve addition and subtraction problems to find unknown angles <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 5

Cluster	Standards as written	
Convert like measurement units within a given measurement system.	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.
Represent and interpret data.	5.MD.B.2	Make a line plot (dot plot) to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in line plot (dot plot). <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i>
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.	5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
	5.MD.C.3a	A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
	5.MD.C.3b	A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and non-standard units.
	5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
	5.MD.C.5a	Find the volume of a right rectangular prism with whole-number edge lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
	5.MD.C.5b	Apply the formula $V = l \times w \times h$ and $V = B \times h$ (where B stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
	5.MD.C.5c	Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

ENTRY POINTS for Measurement and Data Standards in Grade 5

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Convert like measurement units within a given measurement system.	<ul style="list-style-type: none"> Match measurements to corresponding terms and abbreviations for length, weight, and capacity in units <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Convert units of measurement within the same system to solve real-world problems (e.g., convert cm to m) 	<ul style="list-style-type: none"> Use simple measurement conversions to solve real-world problems Convert among different sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Represent and interpret data.	<ul style="list-style-type: none"> Use line plot data involving fractions to answer questions about "more" and "less" <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>		<ul style="list-style-type: none"> Solve problems involving addition and subtraction of fractions by using information presented in line plots <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.	<ul style="list-style-type: none"> Identify parts of a container to determine volume (base, height, length, width) Solve repeated addition problems about volume using technology or manipulatives (e.g., arrange 2 by 3 rectangles of cubes to show $6 + 6 + 6 = 18$ cube units) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Calculate the volume of a rectangular container using manipulatives (e.g., "cubic units") Solve multiplication problems about volume using manipulatives or technology (e.g., arrange 2 by 3 rectangles of cubes to show $6 \times 3 = 18$ cube units) 	<ul style="list-style-type: none"> Solve real-world problems involving volume by building with cubes and applying volume formula <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

Geometry

	Standards	Entry Points	Access Skills
PK	Page 140	Pages 141 – 142	Pages 141 – 142
K	Page 143	Page 144	
1	Page 145	Page 145	
2	Page 146	Page 146	
3	Page 147	Page 147	
4	Page 148	Page 148	
5	Page 149	Page 150	
6	Page 151	Page 152	
7	Page 153	Page 154	
8	Page 155	Pages 156 – 157	

CONTENT AREA Mathematics
DOMAIN Geometry

Pre-Kindergarten		
Cluster	Standards as written	
Identify and describe shapes (squares, circles, triangles, rectangles).	PK.G.A.1	Identify relative positions of objects in space, and use appropriate language (e.g., <i>beside, inside, next to, close to, above, below, apart</i>).
	PK.G.A.2	Identify various two-dimensional shapes using appropriate language.
Analyze, compare, create, and compose shapes.	PK.G.B.3	Create and represent three-dimensional shapes (ball/sphere, square box/cube, tube/cylinder) using various manipulative materials (such as popsicle sticks, blocks, pipe cleaners, pattern blocks).

ENTRY POINTS and ACCESS SKILLS for Geometry Standards in Pre-K

← Less Complex

More Complex →

ACCESS SKILLS

The student will:

Identify and describe shapes (squares, circles, triangles, rectangles).

- ◆ Respond to geometric shapes
- ◆ Track geometric shapes
- ◆ Shift focus from geometric shapes to speaker
- ◆ Shift gaze from speaker to geometric shapes
- ◆ Grasp, release, and/or give geometric shapes
- ◆ Manipulate objects (e.g., move a cube from one hand to the other)
- ◆ Locate geometric shapes partially hidden or out of sight (e.g., remove barrier to expose geometric shapes)
- ◆ Use one object to act on another geometric shape (e.g., use a stapler to attach geometric shapes)
- ◆ Adjust planes to move geometric shapes

The student will:

- ◆ Match identical two-dimensional shapes

ENTRY POINTS

The student will:

- ◆ Identify two-dimensional shapes that are the same (e.g., given a set of different shapes, identify which are triangles, which are circles, etc.)
- ◆ Move given shapes to show location (e.g., given a set of different shapes, move the triangle so it is "next to," "above," "below" the circle)

The student will:

- ◆ Identify shapes by name (e.g., circle, square, triangle, rectangle)
- ◆ Use appropriate language to describe the relative positions of objects in space (e.g., beside, inside, next to, close to, above, below, apart)

ENTRY POINTS and ACCESS SKILLS for Geometry Standards in Pre-K

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>		
	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Identify and describe shapes (squares, circles, triangles, rectangles). (continued)	<ul style="list-style-type: none"> ♦ Turn device on/off to label geometric shapes, (e.g., camera to photograph geometric shapes in the environment) ♦ Imitate action in an activity using geometric shapes ♦ Initiate cause-and-effect response using geometric shapes ♦ Sustain geometry activity through response (e.g., when reading of geometry text stops, vocalize to request more) ♦ Gain attention (e.g., use communication strategy to request a turn or ask for help in an activity utilizing geometric shapes) ♦ Choose from an array of two geometric shapes ♦ Attend visually, aurally, or tactilely to geometric shapes ♦ Given materials to use on templates to create geometric shapes (popsicle sticks, rigid laminated arcs), release the materials onto the template page within 5 seconds of grasping them 			
Analyze, compare, create, and compose shapes.		<ul style="list-style-type: none"> ♦ Identify three-dimensional shapes (e.g., given a set of different shapes, identify which is a sphere) 	<ul style="list-style-type: none"> ♦ Create three-dimensional shapes from manipulatives to match a given similar shaped item (e.g., given a ball to look at, create a sphere from play dough; given a tissue box, create a cube from pipe cleaners) 	<ul style="list-style-type: none"> ♦ Create three-dimensional shapes from manipulatives (e.g., a cube from popsicle sticks, a sphere from play dough)

CONTENT AREA Mathematics
DOMAIN Geometry

Kindergarten

Cluster	Standards as written	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	K.G.A.1	Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as <i>above</i> , <i>below</i> , <i>beside</i> , <i>in front of</i> , <i>behind</i> , and <i>next to</i> .
	K.G.A.2	Correctly name shapes regardless of their orientations or overall size.
	K.G.A.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Analyze, compare, create, and compose shapes.	K.G.B.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
	K.G.B.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
	K.G.B.6	Compose simple shapes to form larger shapes. <i>For example, "Can you join these two triangles with full sides touching to make a rectangle?"</i>

ENTRY POINTS for Geometry Standards in Kindergarten

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	<ul style="list-style-type: none"> ◆ Sort two- and three-dimensional shapes by attribute (e.g., given a group of two-dimensional shapes, sort by shape, , and size.) ◆ Match manipulatives to shapes drawn on paper (e.g., given a paper with drawings of a square, a triangle, a circle, a rectangle, etc. place the matching manipulative on top of the drawing) ◆ Follow directions to make a picture from given two-dimensional shapes (e.g., make a house: start with a rectangle, add a triangle for the roof, add a rectangle for the door, etc.) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify two-dimensional shapes in different sizes by name (e.g., given a group of various shapes, identify all the triangles, or all the squares) ◆ Identify three-dimensional shapes by name (e.g., ball/sphere, square box/cube, tube/cylinder) ◆ Given cards with shapes, follow directions to place in relative positions (e.g., place the triangle beside the square, place the rectangle above the circle, etc.) ◆ Given a group of two-dimensional shapes, place in order according to ordinal numbers (e.g., place the square first, place the triangle second, etc.) 	<ul style="list-style-type: none"> ◆ Differentiate shapes as either two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”) ◆ Match two- and three-dimensional real-life objects with given shapes (e.g., given real-life objects and manipulative shapes, be able to match: ruler/rectangle, clock, circle, ice cream cone/cone, tissue box/cube, etc.) ◆ Given three-dimensional shapes (cube, cone, sphere, etc.), place in relative positions (e.g., place the cone above the cube, place the sphere in front of the cube, etc.) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Analyze, compare, create, and compose shapes.	<ul style="list-style-type: none"> ◆ Indicate one defining attribute of a two-dimensional shape (e.g., round, straight sides, number of sides, number of vertices (corners)) ◆ Match two shapes made from different materials ◆ Distinguish different two-dimensional shapes using sets that are identical (e.g., identify a square given a set of squares and circles that are all the same size) ◆ Distinguish three-dimensional shapes from two-dimensional shapes using manipulatives (e.g., know that squares are different from cubes) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Given a defining attribute, find the two-dimensional shapes with that feature (e.g., find the shapes that have three sides) ◆ Compose simple two-dimensional shapes from manipulatives (e.g., a square from sticks or using technology) ◆ Distinguish different two-dimensional shapes using sets that are identical in size (e.g., organize into sets of each shape when the shapes are all the same size) 	<ul style="list-style-type: none"> ◆ Given a defining attribute, find the two- or three-dimensional shapes with that feature (e.g., choose a quadrilateral when given the attribute “four sides” or choose a cube given the attribute “6 edges equal in length”) ◆ Compose larger shapes from manipulatives or visual models of smaller simple shapes (e.g., compose a rectangle from two squares) ◆ Compose simple three - dimensional shapes from manipulatives (e.g., a sphere from clay or using technology) ◆ Distinguish squares, circles, and triangles using sets that are not identical in size (e.g., organize into sets of each shape when the shapes are not all the same size) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 1

Cluster	Standards as written	
Reason with shapes and their attributes.	1.G.A.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes that possess defining attributes.
	1.G.A.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. ¹
	1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

**ENTRY POINTS for
Geometry Standards in Grade 1**

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Reason with shapes and their attributes.	<ul style="list-style-type: none"> Match shapes to their defining attributes (e.g., find the shape that is round) Identify attributes of different shapes (e.g., which shape has three angles?) Group objects by one defining attribute (e.g., group all shapes with five sides) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Use appropriate language to describe attribute that a group of shapes have in common (e.g., straight lines, round, closed, three-sided,) Identify how many equal parts there are when a shape is divided into 2 or 4 equal parts (e.g., given a circle divided into fourths, say there are 4 equal parts) 	<ul style="list-style-type: none"> Identify defining attributes and non-defining attributes (e.g., all of the rectangles have four sides even if the rectangles are different sizes or different colors) Manipulate equal parts of a shape to compose a whole shape (e.g., given four fourths of a circle, put them together to make a whole circle) Partition (take apart) circles and rectangles into two or four equal shares Use words such as “halves” or “fourths” to describe 2 or 4 equal parts of a shape. Group objects by two defining attributes (e.g., size and number of sides) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

NOTE: Defining attributes mean the shape will always have those characteristics. For example, four right angles is a **defining attribute** of a square.

¹ Students do not need to learn formal names such as “right rectangular prism.”

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 2

Cluster	Standards as written	
Reason with shapes and their attributes.	2.G.A.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. ¹ Identify triangles, squares, rectangles, rhombuses, trapezoids, pentagons, hexagons, and cubes.
	2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i> , <i>thirds</i> , <i>half of</i> , <i>a third of</i> , etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

**ENTRY POINTS for
Geometry Standards in Grade 2**

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Reason with shapes and their attributes	<ul style="list-style-type: none"> Identify different two-dimensional shapes by name (e.g. identify which two-dimensional shapes are triangles) Use appropriate terms to identify shapes by given attribute (e.g., given examples of triangles, quadrilaterals, pentagons, hexagons, and cubes, name the shapes that have five sides) Describe the parts of a rectangle that has been divided into equal parts (e.g., given a rectangle divided into four equal parts, name the parts "fourths") <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Recognize or draw one shape with a specified attribute (e.g. identify a shape with three angles) Use appropriate terms to define shapes by number of sides (e.g., say "triangles have three sides") Match shapes with their defining attributes (e.g., faces, angles) Match same shapes given with different orientations (e.g., given a group of different shapes facing in different directions, identify which shapes belong in each category) Given a rectangle partitioned into rows and columns of equal squares, count the number of squares Partition a rectangle into two, three, or four equal shares 	<ul style="list-style-type: none"> Recognize or draw multiple different shapes with specified attributes (e.g. draw 3 different shapes with four unequal sides) Use appropriate terms to define differences between shapes (e.g., say that "triangles have 3 angles and rectangles have 4 angles" when given a group of both shapes) Sort shapes by number of sides/angles (e.g., given a variety of triangles, quadrilaterals, pentagons, hexagons, sort by number of sides/angles) Partition a rectangle into rows and columns of equal squares. Partition circles and rectangles into two, three, or four equal shares <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

¹ Sizes are compared directly or visually, not compared by measuring.

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 3

Cluster	Standards as written	
Reason with shapes and their attributes.	3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Compare and classify shapes by their sides and angles (right angle/non-right angle). Recognize rhombuses, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
	3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into four parts with equal areas and describe the area of each part as $\frac{1}{4}$ of the area of the shape.</i>

**ENTRY POINTS for
Geometry Standards in Grade 3**

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Reason with shapes and their attributes	<ul style="list-style-type: none"> ◆ Sort two-dimensional shapes by attributes (e.g., length of sides, number of sides) ◆ Compose rectangles from "halves," "thirds," and/or "fourths" (e.g., compose a rectangle given the 4 equal parts) ◆ Decompose rectangles into "halves," "thirds," and/or "fourths" (e.g., given a rectangle, divide it into 2 equal parts by drawing or using manipulatives) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Draw or identify shapes that are and are NOT in different categories (e.g., draw examples of shapes that are squares and those that are NOT squares) ◆ Compose circles from "halves," "thirds," and/or "fourths" (e.g., compose a circle given the 4 equal parts) ◆ Decompose circles into "halves," "thirds," and/or "fourths" (e.g., given a circle, divide it into 2 equal parts by drawing or using manipulatives) 	<ul style="list-style-type: none"> ◆ Draw or identify various shapes that are and are NOT quadrilaterals (e.g., given a group of shapes with different numbers of sides, identify which shapes are quadrilaterals and which are NOT) ◆ Compare shapes by describing their attributes (e.g. given a group of quadrilaterals and pentagons, determine which shapes fall into each category and why) ◆ Identify shapes that have a right angle (e.g., given a set of triangles and various quadrilaterals, identify which shapes have a 90-degree angle) ◆ Name the fraction that describes a part when a shape has been divided into parts with equal area (e.g. given a shape divided into 4 equal parts, know that each part is $\frac{1}{4}$ of the whole shape) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 4

Cluster	Standards as written	
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

**ENTRY POINTS for
Geometry Standards in Grade 4**

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	<ul style="list-style-type: none"> Sort perpendicular or parallel lines based on whether they have those attributes or not (e.g. given drawings of lines that are perpendicular and some that are NOT perpendicular, sort into each category) Sort right angles based on whether they have those attributes or not (e.g. given pictures of angles that measure a variety of degrees, sort into those angles that are right and those that are NOT) Draw a line of symmetry on a given figure that has only one line of symmetry (e.g., draw a line of symmetry on a given picture of a butterfly) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Sort lines by their defining attributes (e.g., given drawings of perpendicular and parallel lines and labels sort into each category) Sort angles by their defining attributes (e.g. given drawings of acute, obtuse, and right angles and labels, sort into each category) Draw/create different angles using manipulatives or drawings (e.g. create right angles out of straws attached by yarn) Draw a line of symmetry on a given figure that has two lines of symmetry (e.g. draw a line of symmetry on a given picture of a rectangle) Identify acute, obtuse, and/or right angles 	<ul style="list-style-type: none"> Identify rays, lines, line segments, and angles (e.g. given a drawing, identify each) Compare rays, lines, line segments, and angles by their attributes (e.g., know that a ray continues on infinitely in one direction because it has an arrow one end only) Identify acute, obtuse, and/or right angles based on their degree measure (e.g. from a drawing know that an acute angle is less than 90 degrees) Identify acute, obtuse, and/or right angles within a shape (e.g., know that a square has four right angles) Distinguish between parallel and non-parallel lines Distinguish between perpendicular and non-perpendicular lines Identify all lines of symmetry on a given figure (e.g. given a drawing of a figure with several lines, identify which lines are lines of symmetry) Identify angles in diagrams as acute, obtuse, right, or straight <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 5

Cluster	Standards as written	
Graph points on the coordinate plane to solve real-world and mathematical problems.	5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x -axis and x -coordinate, y -axis and y -coordinate).
	5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.
Classify two-dimensional figures into categories based on their properties.	5.G.B.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i>
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties. <i>For example, all rectangles are parallelograms because they are all quadrilaterals with two pairs of opposite sides parallel.</i>

ENTRY POINTS for Geometry Standards in Grade 5

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Graph points on the coordinate plane to solve real-world and mathematical problems.	<ul style="list-style-type: none"> ◆ Label components of the coordinate system (e.g., origin, ordered pair, x-axis, y-axis) ◆ Determine the distance from zero a given point is on a number line, given a point on a number line (e.g. what is the distance from zero to a point graphed at 9?) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Plot a point to show the location of whole numbers on a number line (e.g., locate a temperature on a diagram of a thermometer) ◆ Determine the distance traveled along the x-axis and y-axis from the origin given a point in the first quadrant of a labeled coordinate plane (e.g., the point (3, 2) is graphed 3 units from the origin along the x-axis and 2 units from the origin along the y-axis) 	<ul style="list-style-type: none"> ◆ Graph ordered pairs at whole number intervals in the first quadrant of the coordinate plane (e.g., graph the point (3, 2)) ◆ Interpret real-world mathematical problems by plotting points on a coordinate plane (e.g., identify a possible location to graph a point that is 3 units from the location of the “swings at the park” – more than one correct answer) ◆ Identify coordinates of points in the first quadrant of the coordinate plane ◆ Identify coordinates of points to solve real-world or mathematical problems (e.g., identify the coordinates where the “school” is located on a coordinate plane) ◆ Graph ordered pairs in any quadrant of the coordinate plane <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Classify two-dimensional figures into categories based on their properties.	<ul style="list-style-type: none"> ◆ Identify given shapes presorted into subcategories determine what they have in common (e.g., rectangles, parallelograms and trapezoids are all four-sided shapes) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify shapes by attributes (e.g., by number of sides/angles or by number of parallel/congruent sides) ◆ Label congruent sides and angles within a parallelogram 	<ul style="list-style-type: none"> ◆ Classify quadrilaterals in a hierarchy based on properties ◆ Describe figures using properties of sides and angles <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 6

Cluster	Standards as written	
Solve real-world and mathematical problems involving area, surface area, and volume.	6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
	6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
	6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface areas of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

ENTRY POINTS for Geometry Standards in Grade 6

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Solve real-world and mathematical problems involving area, surface area, and volume.	<ul style="list-style-type: none"> ◆ Decompose a special quadrilateral (e.g., kite, parallelogram, etc.) into triangles and rectangles ◆ Distinguish figures that would have volume from those that don't (e.g., three-dimensional vs. two-dimensional figures) ◆ Calculate the area of a two-dimensional figure using manipulatives (e.g., count the number of unit squares that are contained in a rectangle) ◆ Calculate the surface area of a right rectangular prism using manipulatives (e.g., count the number of unit squares contained in each face) ◆ Calculate the volume of a right rectangular prism by decomposing into unit cubes (e.g., for prisms with integer edge-lengths) ◆ Calculate the volume of a right rectangular prism using the formula (e.g., using multiplication) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Find the area of a parallelogram by composing into a rectangle and triangles (e.g., using subtraction) ◆ Find the area of a parallelogram by decomposing into a rectangles and triangles (e.g., using addition) ◆ Match a three-dimensional figure and its appropriate net composed of rectangles and triangles ◆ Find the volume of a right rectangular prism packed with unit cubes with fractional edge lengths (e.g., $\frac{1}{2}$, $\frac{1}{3}$, etc.) ◆ Calculate the length of a line segment on a coordinate grid (e.g., with endpoints with the same x- or y-coordinates) ◆ Calculate the area of a two-dimensional figure (square, rectangle, triangle, etc.) given a formula 	<ul style="list-style-type: none"> ◆ Find the area of triangles and parallelograms by composing into rectangles and right triangles (e.g., using subtraction) ◆ Solve real-life problems using the area formula (e.g., for squares, rectangles, triangles, etc.) ◆ Calculate the perimeter of a rectangle on a coordinate grid by using coordinates of the vertices (e.g., with same x- and y-coordinates) ◆ Find the volume of right rectangular prisms with fractional side lengths in two ways, by packing it with cubes and by using the volume formula ◆ Draw a line segment of a given length on a coordinate grid (e.g., using coordinates) ◆ Use the net of a figure with labeled dimensions made up of rectangles and triangles (e.g., prism, pyramid) to calculate its surface area <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 7

Cluster	Standards as written	
Draw, construct, and describe geometrical figures and describe the relationships between them.	7.G.A.1	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
	7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
	7.G.A.3	Describe the shape of the two-dimensional face of the figure that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	7.G.B.4	Circles and measurement:
	7.G.B.4.a	Know that a circle is a two-dimensional shape created by connecting all of the points equidistant from a fixed point called the center of the circle
	7.G.B.4.b	Understand and describe the relationships among the radius, diameter, circumference and circumference of a circle
	7.G.B.4.c	Understand and describe the relationship among the radius, diameter, and area of a circle
	7.G.B.4.d	Know the formulas for the area and circumference of a circle and use them to solve problems
	7.G.B.4.e	Give an informal derivation of the relationship between the circumference and area of a circle
	7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write simple equations and use them to solve for an unknown angle in a figure
	7.G.B.6	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

ENTRY POINTS for Geometry Standards in Grade 7

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Draw, construct, and describe geometrical figures and describe the relationships between them.	<ul style="list-style-type: none"> ◆ Construct a triangle given one angle measure using tools (e.g., construct a triangle that has an angle measure of 50°) ◆ Construct triangles given one angle measure using tools (e.g., construct four different triangles that each have an angle measure of 75°) ◆ Construct a geometric shape with given conditions (e.g., draw a rectangle with two sides of two inches, and two sides of one inch) ◆ Construct a circle of a given radius using tools (e.g., a compass) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Construct a triangle given three side lengths using tools (e.g., ruler, compass, computer software, etc.) ◆ Reproduce a two-dimensional geometric figure at a different scale using tools (e.g., ruler, compass, computer software, etc.) ◆ Distinguish between a three side-length combination that cannot form a triangle and one that can (e.g., side lengths of 3, 5, and 9 units cannot make a triangle, but side lengths of 7, 7, and 8 units can) 	<ul style="list-style-type: none"> ◆ Solve a real-life problem using a scale drawing (e.g., a word problem) ◆ Construct a triangle given two side lengths and the included angle using tools (e.g., ruler, protractor) ◆ Construct a right triangle given the lengths of the hypotenuse and one leg (e.g., draw the right triangle that has a leg of 4 units and a hypotenuse of 7 units) ◆ Describe the shape that results from a specific slice in a solid object (e.g., a vertical slice in a right square prism that includes a diagonal of a base) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	<ul style="list-style-type: none"> ◆ Identify radius, diameter, and center of a circle ◆ Calculate the diameter of a circle given its radius ◆ Calculate the area of a circle given its radius ◆ Identify (labeled) vertical or supplementary angles in a diagram of two intersecting lines (e.g., a and c are vertical angles and angles b and c are supplementary) ◆ Determine an angle's complement and/or supplement (e.g., given an angle of 29°, its complement is 61° and its supplement is 151°) ◆ Calculate the radius of circle, given its diameter <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Calculate the diameter and circumference of a circle given its radius ◆ Identify vertical, adjacent, complementary and supplementary angles in a diagram with intersecting lines (e.g., with labeled angles) ◆ Calculate the surface area of a cube in a real-life or mathematical situation 	<ul style="list-style-type: none"> ◆ Calculate the diameter, circumference, and area of a circle given its radius ◆ Solve a real-life problem involving circumference and area of a circle ◆ Calculate the surface area of a right rectangular prism in a real-life or mathematical situation ◆ Calculate the volume of a right rectangular prism given its dimensions in a real-life or mathematical situation ◆ Calculate a missing dimension of a right rectangular prism given its volume (e.g., given volume, length, and width, find the height) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 8

Cluster	Standards as written	
Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: (1a, 1b, 1c)
	8.G.A.1a	Lines are transformed to lines, and line segments to line segments of the same length.
	8.G.A.1b	Angles are transformed to angles of the same measure.
	8.G.A.1c	Parallel lines are transformed to parallel lines.
	8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them.
	8.G.A.3	Describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
	8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <i>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</i>
Understand and apply the Pythagorean Theorem.	8.G.B.6a	Understand the relationship among the sides of a right triangle.
	8.G.B.6b	Analyze and justify the Pythagorean Theorem and its converse using pictures, diagrams, narratives, or models
	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions
	8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

ENTRY POINTS for Geometry Standards in Grade 8

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand congruence and similarity using physical models, transparencies, or geometry software.	<ul style="list-style-type: none"> ◆ Demonstrate a reflection, translation, and rotation of a two-dimensional figure using manipulatives ◆ Demonstrate a reflection, translation, and rotation of a two-dimensional figure using manipulatives or technology on a coordinate plane (i.e. in multiple ways) ◆ Reflect a figure over the x- and y-axis (e.g., on a coordinate grid) ◆ Record movement resulting from a translation by counting to measure how many spaces each of the vertices of a figure moved left, right, up or down on a coordinate grid ◆ Demonstrate which angles are congruent in a diagram showing two parallel lines cut by a transversal by using and comparing angle wedges ◆ Identify an image as a reflection, rotation, and a translation of a pre-image of a two-dimensional figure (e.g., using manipulatives, technology, a coordinate grid, etc.) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Translate a figure a given number of units up, down, left, or right on a coordinate grid ◆ Reflect a figure over the x- and y-axis on a coordinate grid ◆ Rotate a figure 90° about the origin on a coordinate grid (e.g., clockwise and/or counterclockwise) ◆ Dilate a figure given a scale factor with respect to the origin on a coordinate grid (e.g., dilate a triangle with scale factor of 2) ◆ Calculate a missing angle measure in a triangle given the measures of the other two angles ◆ Calculate a missing angle measure in a diagram showing two parallel lines cut by a transversal (e.g., using corresponding, alternate exterior, vertical angles, etc.) ◆ Identify congruent angles in a diagram with parallel lines cut by a transversal (e.g., vertical, alternate interior, corresponding angles, etc.) ◆ Identify supplementary angles in a diagram with parallel lines cut by a transversal (e.g., linear pairs, same side interior angles, etc.) 	<ul style="list-style-type: none"> ◆ Reflect a figure over a given line on a coordinate grid (e.g., over an axis, $x = 2$, $y = x$, etc.) ◆ Rotate a figure any number of degrees about the origin on a coordinate grid (e.g., rotate 270° clockwise about the origin) ◆ Dilate a figure given a scale factor with respect to any point on a coordinate grid (e.g., use a vertex of the figure as the center of dilation) ◆ Demonstrate a sequence of transformations that exhibits the congruence of two figures on a coordinate grid ◆ Calculate a missing angle measure in a triangle given variable expressions that represent the measures of the other two angles ◆ Label all angles as either congruent or supplemental in a diagram with parallel lines cut by a transversal ◆ Find the sum of the measures of the interior angles of any polygon (e.g., by dividing into triangles) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Geometry Standards in Grade 8

← Less Complex

More Complex →

Understand and apply the Pythagorean Theorem.	<ul style="list-style-type: none"> Identify right triangles (e.g., distinguish right triangles from acute and obtuse triangles) Label the components of a right triangle (e.g., legs, hypotenuse, right angle) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Apply the Pythagorean Theorem to find a missing side length in a right triangle (e.g., from a diagram and/or a description) Apply the Pythagorean Theorem to find the distance between points on a coordinate grid (e.g., given coordinates, a graph, etc.) 	<ul style="list-style-type: none"> Apply the Pythagorean Theorem in real world situations to find a missing length in a naturally occurring right triangle (e.g., length of a ladder) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	<ul style="list-style-type: none"> Choose the correct formula necessary to calculate volume of a given solid object (e.g., choose the volume formula for a sphere to calculate the volume of a basketball) Use the formula to calculate the volume of a prism (e.g., as a real-life object or simply given the dimensions) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> Use the formula to calculate the volume of a cylinder (e.g., as a real-life object or simply given the dimensions) Use the formula to calculate the volume of a pyramid (e.g., as a real-life object or simply given the dimensions) Use the formula to calculate the volume of a cone (e.g., as a real-life object or simply given the dimensions) Use the formula to calculate the volume of a sphere (e.g., as a real-life object or simply given the dimensions) 	<ul style="list-style-type: none"> Use the formula to calculate the volume of any given solid object (e.g., as a real-life object or simply given the dimensions) Use the volume of a solid object to find a missing dimension of any solid object (e.g., prism, cylinder, cone, pyramid, and/or sphere) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

Statistics and Probability

	Standards	Entry Points	Access Skills
6	Page 159	Page 160 – 161	Pages 160 – 162
7	Pages 163 – 164	Pages 165 – 166	
8	Page 167	Page 168	

CONTENT AREA Mathematics
DOMAIN Statistics and Probability

Grade 6

Cluster	Standards as written	
Develop understanding of statistical variability.	6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <i>For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.</i>
	6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median, mean, and/or mode), spread (range, interquartile range), and overall shape.
	6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Summarize and describe distributions.	6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
	6.SP.B.4a	Read and interpret circle graphs.
	6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: (5a, 5b, 5c, 5d)
	6.SP.B.5a	Reporting the number of observations.
	6.SP.B.5b	Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
	6.SP.B.5c	Giving quantitative measures of center (median, and/or mean) and variability (range and/or interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
	6.SP.B.5d	Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

ENTRY POINTS and ACCESS SKILLS for Statistics and Probability Standards in Grade 6

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>	
	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Develop understanding of statistical variability.	<ul style="list-style-type: none"> ◆ Turn on technology used to collect data (e.g., turn on computer to gather information) ◆ Imitate action used to conduct survey (e.g., imitate classmate to distribute survey) ◆ Sustain a survey activity through response (e.g., reach out to shake hands to approach a survey respondent) ◆ Gain attention to conduct a survey (e.g., raise hand vocalize) 	<ul style="list-style-type: none"> ◆ Draw a simple conclusion from a data display (e.g., more students in class are less than 5 feet tall than are greater than 5 feet tall) 	<ul style="list-style-type: none"> ◆ Represent data from a survey graphically (e.g., in a table or other data display) ◆ Draw conclusions from the comparison of two sets of data (e.g., more students have brown eyes than have blue eyes)
Summarize and describe distributions.	<ul style="list-style-type: none"> ◆ Respond to materials to create tables, charts, or graphs ◆ Track materials used to create tables, charts, or graphs ◆ Shift focus from materials used to create tables, charts, or graphs to speaker ◆ Grasp materials used to create tables, charts, or graphs ◆ Use two hands to hold materials used to create tables, charts, or graphs 	<ul style="list-style-type: none"> ◆ Distinguish between a variety of data displays (e.g., by matching, word bank, etc.) ◆ Answer basic questions about data displayed in a dot plot (e.g., Which has more/less?) ◆ Order a set of numerical data, based on an unordered list of data (e.g. record the daily high temperature for two weeks and order the temperatures from least to greatest) 	<ul style="list-style-type: none"> ◆ Answer questions related to single variable data shown in a data display (e.g., a dot plot, bar graph, circle graph, histogram, box plot, etc.) ◆ Calculate the mean for a set of data (e.g., from a dot plot, a list of numerical data, etc.) ◆ Locate the median of a set of data (e.g., in a histogram, frequency table, etc.) ◆ Match data displays to descriptions of observations (e.g., choose a display that shows a correct distribution of data)
			<ul style="list-style-type: none"> ◆ Sort questions that are statistical from those that are not (e.g., "What is your favorite color?" is statistical and "What color is this?" is not)
			<ul style="list-style-type: none"> ◆ Identify the measures of center for a set of numerical data (e.g., for a list of data find the mean, median, and/or mode) ◆ Give a five-number summary for a set of data (e.g., displayed in a box plot) ◆ Select an appropriate display for a set of data (e.g., for data that describes parts of a whole, a circle graph is appropriate) ◆ Equate sets of numerical data with corresponding data displays (e.g. by matching)

ENTRY POINTS and ACCESS SKILLS for Statistics and Probability Standards in Grade 6

← **Less Complex**

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>
Summarize and describe distributions. (continued)	<ul style="list-style-type: none"> ◆ Release materials used to create tables, charts, or graphs ◆ Move materials used to create tables, charts, or graphs ◆ Orient materials used to create tables, charts, or graphs ◆ Manipulate objects used to create tables, charts, or graphs ◆ Locate objects partially hidden or out of sight needed to create tables, charts, or graphs (e.g., remove barrier to expose materials) ◆ Use one object to act on another to create tables, charts, or graphs (e.g., use scissors to cut materials) ◆ Turn on/off technology used to create tables, charts, or graphs (e.g., turn on voice-generating device to turn on technology tool to create graph program) ◆ Imitate action used to create tables, charts, or graphs (e.g., imitate classmate attaching icon to graph) 	<ul style="list-style-type: none"> ◆ Calculate the range of a set of data (e.g., by finding the difference of the maximum and minimum values in the list of data) ◆ Organize data collected on two characteristics on a template (e.g., boys' and girls' seasons of birth) ◆ Create a graph to represent a data set 	<ul style="list-style-type: none"> ◆ Identify the median of a data set (e.g., from a dot plot, box plot, ordered list, unordered list, etc.) ◆ Draw a scaled bar graph (e.g., each interval represents the number of students' pets) ◆ Answer questions using information presented on a data display (e.g., a bar graph, a line graph, etc.)

ACCESS SKILLS (continued) for Statistics and Probability Standards in Grade 6

← Less Complex

More Complex →

ACCESS SKILLS The student will:

ENTRY POINTS The student will:

Summarize
and describe
distributions.
(continued)

- ◆ Initiate cause-and-effect response by turning on technology tool to activate graphing computer program
- ◆ Sustain graphing activity through response
- ◆ Gain attention in a graphing activity (e.g., raise hand vocalize)
- ◆ Make a request during graphing relationship activity (e.g., request a turn)
- ◆ Choose materials to be used in graphing activity
- ◆ Attend visually, aurally, or tactilely to materials to create tables, charts, or graphs

CONTENT AREA Mathematics
DOMAIN Statistics and Probability

Grade 7

Cluster	Standards as written	
Use random sampling to draw inferences about a population.	7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; Generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
	7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. <i>For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</i>
Draw informal comparative inferences about two populations.	7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team and both distributions have similar variability (mean absolute deviation) of about 5 cm. The difference between the mean heights of the two teams (10 cm) is about twice the variability (5 cm) on either team. On a dot plot, the separation between the two distributions of heights is noticeable.</i>
	7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. <i>For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</i>
Investigate chance processes and develop, use, and evaluate probability models.	7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i>
	7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
	7.SP.C.7a	Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. <i>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</i>

evaluate probability models. (continued)	7.SP.C.7b	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. <i>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</i>
	7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
	7.SP.C.8a	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
	7.SP.C.8b	Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
	7.SP.C.8c	Design and use a simulation to generate frequencies for compound events. <i>For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least four donors to find one with type A blood?</i>

ENTRY POINTS for Statistics and Probability Standards in Grade 7

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Use random sampling to draw inferences about a population.	<ul style="list-style-type: none"> ◆ Interpret graphical representations of data (e.g., number of people in sample, information collected) ◆ Collect, represent and/or interpret the graphical representations of data <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Draw conclusions from a data sample represented graphically (e.g., least/greatest) 	<ul style="list-style-type: none"> ◆ Explain how a random survey sample could be obtained from a larger population set (e.g., every other person vs. every fifth person in a class list) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Draw informal comparative inferences about two populations.	<ul style="list-style-type: none"> ◆ Interpret two sets of numerical data (e.g., there are more players on the football team (set 1) than there are on the basketball team (set 2)) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Draw conclusions from the comparison of two sets of numerical data (e.g., more students have brown eyes than blue eyes) ◆ Compare data from a local survey to a published norm (e.g., class distribution of men vs. women to national statistics) 	<ul style="list-style-type: none"> ◆ Draw conclusions from the comparison of two sets of numerical data using measures of center (e.g., mean height of members of the football team and members of the basketball team) ◆ Compare data from a random sample vs. a non-random sample (e.g., compare the selection of favorite songs of a random sample of the school vs. your best friends) ◆ Determine the most accurate measure of center from examining data sets <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Investigate chance processes and develop, use, and evaluate probability models.	<ul style="list-style-type: none"> ◆ Record the outcomes in a simple probability experiment (e.g., choosing a red marble from a bag of assorted marbles) ◆ Use models to calculate the probability of an event (e.g., probability of landing on the green section of a spinner) 	<ul style="list-style-type: none"> ◆ Determine that the probability of an event occurring is likely, unlikely, certain, or impossible ◆ Interpret the outcomes of a probability experiment with different number of trials ◆ Explain predictions of the outcomes of a probability experiment (e.g., getting an even number on a number cube) 	<ul style="list-style-type: none"> ◆ Describe the probability of an event occurring as a number between 0 (impossible) and 1 (certain) ◆ Compare actual results of a simple experiment with theoretical probabilities using models and tables

ENTRY POINTS for Statistics and Probability Standards in Grade 7

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Investigate chance processes and develop, use, and evaluate probability models. (continued)	<i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i>	♦ Select possible outcomes of a compound event from cards (e.g., if footwear (sneakers, flats, heels) is available in red and blue, select from cards that give pictures of all possible outcomes; i.e., heels in red, heels in blue, sneakers in red, sneakers in blue, flats in red, flats in blue)	♦ Organize (list/show/draw) all possible outcomes for compound events (e.g., if footwear: sneakers, flats and heels is available in colors: red and blue, list all possible selections – sneakers in red, flats in red, heels in red, sneakers in blue, heels in blue, flats in blue) <i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i>

CONTENT AREA Mathematics
DOMAIN Statistics and Probability

Grade 8

Cluster	Standards as written	
Investigate patterns of association in bivariate data.	8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i>
	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. <i>For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</i>

ENTRY POINTS for Statistics and Probability Standards in Grade 8

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	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Investigate patterns of association in bivariate data.	<ul style="list-style-type: none"> ◆ Represent numerical data on a scatter plot (e.g., plot ordered pairs that represent numerical data) ◆ Identify the bivariate data represented by points on a scatter plot (e.g., for a scatter plot that displays age and height of students, identify, for the ordered pair (15,60), a height of 60 inches for a 15-year-old student) ◆ Determine whether data suggests a linear relationship (e.g., classify data as linear or not linear by inspecting a scatter plot) ◆ Draw a line of best fit for data displayed in a scatter plot (e.g., by using a ruler to draw a line through the points) ◆ Describe tendencies displayed by data in a scatter plot (e.g., show that a greater number of hours of study results in higher test scores) <p><i>See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Describe the nature of the association of linear data displayed on a scatter plot (e.g., describe data as increasing, decreasing, neither increasing nor decreasing (constant), positive association, negative association, no correlation, etc.) ◆ Analyze frequencies in a two-way table to describe tendencies (e.g., students who have pets also check out books more about animals from the library than students who don't have pets) ◆ Describe tendencies displayed by data in a scatter plot (e.g., show that a greater number of hours of study results in higher test scores) ◆ Complete a two-way table summarizing data on two categorical variables from the same subjects (e.g., fill in missing data, including totals, in a table presenting survey data from 20 people asking if they had dogs and if they had allergies) 	<ul style="list-style-type: none"> ◆ Describe the strength of the association of linear data displayed on a scatter plot (e.g., describe data as having strong positive association, weak negative association, no association, etc.) ◆ Use the line of best fit on a scatter plot of data to predict likely outcomes (e.g., plant growth, rainfall) ◆ Interpret the slope from a linear model of bivariate data (e.g., for a scatter plot that shows temperature vs. ice cream sales, interpret the slope as the increase in ice cream sales as the temperature rises) ◆ Interpret the y-intercept from a linear model of bivariate data (e.g., for a scatter plot that shows test scores vs. days absent, interpret the y-intercept as the test score received if missing no days of school) ◆ Analyze a two-way table summarizing data on two categorical variables from the same subject (e.g., by answering quantitative questions and/or questions about association) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

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High School Conceptual Category – Number and Quantity

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CONTENT AREA Mathematics

CONCEPTUAL CATEGORY Number and Quantity

DOMAIN The Real Number System

High School

Cluster	Standards as written	
Extend the properties of exponents to rational exponents.	H.N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5$ $(1/3)^3$ to hold, so $(5^{1/3})^3$ must equal 5.</i>
	H.N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Use properties of rational and irrational numbers.	H.N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

ENTRY POINTS and ACCESS SKILLS for Number and Quantity

The Real Number System Standards in High School

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	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Extend the properties of exponents to rational exponents.	<ul style="list-style-type: none"> ◆ Respond to materials as they are counted ◆ Shift focus from materials to speaker counting materials ◆ Grasp materials as they are counted ◆ Release materials as they are counted ◆ Give materials as they are counted ◆ Move objects as they are counted ◆ Orient objects as they are counted (e.g., turn flowerpots upright) ◆ Manipulate objects with two hands as they are counted ◆ Locate objects partially hidden or out of sight to add to a collection of objects to be counted ◆ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ◆ Use two hands to manipulate objects to be counted ◆ Functionally use materials in a counting activity ◆ Activate device in a counting activity ◆ Imitate action in a counting activity 	<ul style="list-style-type: none"> ◆ Match the figure of a square with given dimensions to the area of the square (e.g., match a square with side lengths of 5 units to an area of 25 square units) ◆ Illustrate a representation of a cube using manipulatives or arrays (e.g. a Rubik's Cube has dimensions $3 \times 3 \times 3$, or choose the cubic array from a group of arrays shaped like non-cubic prisms) 	<ul style="list-style-type: none"> ◆ Rewrite whole number expressions of repeated multiplication using exponents, or rewrite exponent expressions using whole numbers representing repeated multiplication (e.g., $2 \times 2 \times 2 = 2^3$ and/or $6^4 = 6 \times 6 \times 6 \times 6$) ◆ Evaluate numbers or expressions written with exponents (e.g., 4^2 is equal to 16 and/or $(2x)^2$ is equal to $4x^2$) ◆ Illustrate the reciprocal relationship between a perfect square and a square root (e.g., $\sqrt{25} = 5$ and $5^2 = 25$) ◆ Identify perfect squares within 1000 (e.g., from a list of 200, 300, 400, and 500, the perfect square is 400) ◆ Write or evaluate an expression that represents the area of a square using exponents (e.g., $6 \times 6 = 6^2 = 36$) 	<ul style="list-style-type: none"> ◆ Illustrate the reciprocal relationship between a square and a square root for integers (e.g., $(\sqrt{7})^2 = 7$ and $(\sqrt{16})^2 = 4^2 = 16$) ◆ Identify perfect squares and perfect cubes within 1000 (e.g., from a list of 4, 24, 44, and 64, 4 is a perfect square and 64 is both a perfect square and a perfect cube) ◆ Represent numbers using scientific notation (e.g., $432 = 4.32 \times 10^2$) ◆ Solve problems using square roots (e.g., the side length of a square rug with an area of 20 ft^2 is $\sqrt{20}$ feet) ◆ Demonstrate, with integers or variables, the properties of exponents such as the Product Rule (e.g., $2^3 \cdot 2^2 = 2^5$), the Quotient Rule (e.g., $\frac{x^5}{x^2} = x^3$), or the Power Rule (e.g., $(7^2)^4 = 7^8$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ACCESS SKILLS (continued) for Number and Quantity
The Real Number System Standards in High School

Less Complex

More Complex

<u>ACCESS SKILLS</u> <u>The student will:</u>		<u>ENTRY POINTS</u> <u>The student will:</u>
Extend the properties of exponents to rational exponents. (continued)	<ul style="list-style-type: none"> ◆ Locate partially hidden or out of sight objects or materials in a counting activity ◆ Construct or assemble materials in a counting activity ◆ Initiate cause and effect response in a counting activity ◆ Sustain activity through response in a counting activity ◆ Gain attention in a counting activity ◆ Make a request in a counting activity ◆ Choose from an array of errorless choices in a counting activity ◆ Use one object to act on another in a counting activity (e.g., use a pointer to tap) ◆ Attend visually aurally, or tactilely to materials as they are counted 	

ENTRY POINTS and ACCESS SKILLS for Number and Quantity

The Real Number System Standards in High School

← Less Complex

More Complex →

ACCESS SKILLS		ENTRY POINTS	
The student will:	The student will:	The student will:	The student will:
Use properties of rational and irrational numbers.	(Refer to Access Skills above)	<ul style="list-style-type: none"> Sequence fractions or mixed numbers and integers in a list or on a number line (e.g., $\frac{4}{7}$ comes before 2, which comes before $3\frac{2}{5}$) Multiply and divide fractions (e.g., $\frac{1}{3} \cdot \frac{3}{5} = \frac{3}{15}$ or $\frac{1}{3} \div \frac{3}{5} = \frac{5}{9}$) Add and subtract fractions with like or unlike denominators (e.g., $\frac{6}{5} - \frac{2}{5} = \frac{4}{5}$ or $\frac{1}{3} + \frac{2}{6} = \frac{4}{6}$) Identify rational numbers (e.g., $\frac{6}{13}$ is a rational number because it is a ratio of two integers) or irrational numbers (e.g., from a list of numbers, identify π and $\sqrt{6}$ as irrational numbers) 	<ul style="list-style-type: none"> Estimate the location of irrational numbers on a number line (e.g., $\sqrt{23}$ lies between 4 and 5 on a number line, but closer to 5 since 23 is closer to 25 than 16) Estimate square roots of integers (e.g., $\sqrt{15}$ is a little less than 4, because $4^2 = 16$) Categorize rational and irrational numbers within the real number system (e.g., use a Venn diagram to illustrate to which each number in a list belongs) Determine whether a number is rational by showing that it can be written as a repeating or terminating decimal (e.g., $\frac{7}{3}$, $\frac{471}{100}$, and $\sqrt{25}$ are rational because they are equal to 2.$\overline{33}$, 4.71, and 5, but $\sqrt{10}$ is irrational)

Continue to address skills and concepts that approach grade-level expectations in this cluster

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Number and Quantity
DOMAIN Quantities

High School

Cluster	Standards as written	
Reason quantitatively and use units to solve problems.	H.N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *
	H.N-Q.2	Define appropriate quantities for the purpose of descriptive modeling. *
	H.N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *
	H.N-Q.MA3a	Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. *

* indicates Modeling standard

ENTRY POINTS and ACCESS SKILLS for Number and Quantity Quantities Standards in High School

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Reason quantitatively and use units to solve problems.	<ul style="list-style-type: none"> ◆ Shift focus from materials to speaker counting materials or measured ◆ Grasp materials as they are counted or measured ◆ Release materials as they are counted or measured ◆ Give materials as they are counted or measured ◆ Move objects as they are counted or measured ◆ Orient objects as they are counted or measured (e.g., turn flowerpots upright) ◆ Manipulate objects with two hands as they are counted or measured ◆ Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted or measured 	<ul style="list-style-type: none"> ◆ Describe the relationship of two quantities by interpreting rate (e.g., show that a rate given in miles per hour shows the relationship of distance and time) ◆ Compare part-to-whole relationships (e.g., if, out of a total of 12 pets 5 are cats, then $\frac{5}{12}$ of the pets are cats) ◆ Compare part-to-part relationships (e.g., if there are 4 dolphins and 5 whales, then the ratio of dolphins to whales is represented by 4 : 5) ◆ Round numbers to a specific place value in real-life and/or mathematical problems (e.g., 14,375 rounded to the nearest thousand is 14,000, but rounded to the nearest hundred is 14,400) 	<ul style="list-style-type: none"> ◆ Determine a suitable unit rate for a solution of a real-world problem (e.g., a suitable rate for a car on a trip is miles per day, but for a thrown ball is feet per second) ◆ Use rounding strategies to make simple estimates (e.g., a play with a duration of 115 minutes lasted about 2 hours) ◆ Choose an appropriate unit of measurement for a real-life problem (e.g., which would best describe the weight of a dog: grams, inches, kilograms, or tons?) ◆ Use rounding strategies to make estimates in real-life or mathematical problems (e.g., to purchase 5 gallons of milk which cost \$3.35 each, one would need less than \$20) 	<ul style="list-style-type: none"> ◆ Perform conversions of units between systems or scales of measurements (e.g., 3 kilograms is equal to 3000 grams, or 13,200 feet is equal to 2½ miles) ◆ Solve proportions in real-life or mathematical situations where one quantity is unknown (e.g., $\frac{5}{3} = \frac{x}{9}$ or if you can buy 6 avocados for \$3, how many can you buy with \$12?) ◆ Determine the unit rate in real-life situations (e.g., if 4 cans of tomato soup cost \$5, what is the cost per can of the soup?) ◆ Express the solutions of real-life measurement problems using the appropriate unit rate (e.g., a car that travels 80 miles in two hours averages 40 miles per hour, or a student who took 45 minutes to complete 15 homework problems worked at a rate of 3 minutes per problem)
				<i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i>

ACCESS SKILLS (continued) for Number and Quantity Quantities Standards in High School

← Less Complex

More Complex →

ACCESS SKILLS The student will:

ENTRY POINTS The student will:

Reason
quantitative-
ly and use
units to
solve
problems.
(continued)

- ◆ Use one object to act on another as objects are counted or measured (e.g., use a pointer to tap, hold measurement tool against object to be measured)
- ◆ Adjust plane to move objects in counting activities or measurement (e.g., tip plank so that materials can be named in counting sequence as they fall)
- ◆ Construct using materials that have been counted in sequence or measured (e.g., tower of blocks)
- ◆ Turn device on/off to participate in counting sequence or measurement activity (e.g., activate preprogrammed voice-generating device to recite number names)
- ◆ Imitate action in counting sequence or measurement activity
- ◆ Initiate cause-and-effect response in counting sequence or measurement activity (e.g., use switch to activate a number-naming cause-and-effect computer program)
- ◆ Sustain counting sequence activity through response
- ◆ Gain attention in counting sequence or measurement activity
- ◆ Make a request in counting sequence or measurement activity (e.g., request a turn to move the marker on a board game)
- ◆ Choose from an array of two during a counting sequence or measurement activity (e.g., choose materials to be counted or measured)
- ◆ Choose beyond an array of two during a counting sequence or measurement activity (e.g., choose materials to be counted or measured)
- ◆ Follow directions in counting sequence or measurement activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts)
- ◆ Track object as it is added or subtracted from a set of objects
- ◆ Attend visually, aurally, or tactilely to materials as they are counted or measured

CONTENT AREA Mathematics

CONCEPTUAL CATEGORY Number and Quantity

DOMAIN The Complex Number System

High School

Cluster	Standards as written	
Perform arithmetic operations with complex numbers.	H.N-CN.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
	H.N-CN.2	Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.
	H.N-CN.3	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane.	H.N-CN.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
	H.N-CN.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.</i>
	H.N-CN.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Use complex numbers in polynomial identities and equations.	H.N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.
	H.N-CN.8	(+) Extend polynomial identities to the complex numbers. <i>For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.</i>
	H.N-CN.9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

+ indicates standard is beyond College and Career Ready

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

ENTRY POINTS and ACCESS SKILLS for Number and Quantity

The Complex Number System Standards in High School

Less Complex

More Complex

Standards labeled with a (+) symbol appear in courses intended for all students but are considered “beyond College and Career Ready” and will not be assessed.

Perform arithmetic operations with complex numbers.

These standards are not common to both high school pathways and will not be assessed.

Represent complex numbers and their operations on the complex plane.

These standards are considered “beyond College and Career Ready” and will not be assessed.

Use complex numbers in polynomial identities and equations.

These standards are not common to both high school pathways and will not be assessed.

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Number and Quantity
DOMAIN Vector and Matrix Quantities

High School

Cluster	Standards as written	
Represent and model with vector quantities.	H.N-VM.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $ \mathbf{v} $, $\ \mathbf{v}\ $, v).
	H.N-VM.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
	H.N-VM.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors.	H.N-VM.4	(+) Add and subtract vectors.
	H.N-VM.4a	(+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that (+) the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
	H.N-VM.4b	(+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
	H.N-VM.4c	(+) Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w} , with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	H.N-VM.5	(+) Multiply a vector by a scalar.
	H.N-VM.5a	(+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
	H.N-VM.5b	(+) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\ = c \mathbf{v} $. Compute the direction of $c\mathbf{v}$ knowing that when $ c \mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).

+ indicates standard is beyond College and Career Ready

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

Perform operations on matrices and use matrices in applications.	H.N-VM.6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
	H.N-VM.7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
	H.N-VM.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
	H.N-VM.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a Commutative operation, but still satisfies the Associative and Distributive properties.
	H.N-VM.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	H.N-VM.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
	H.N-VM.12	(+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

+ indicates standard is beyond College and Career Ready

ENTRY POINTS and ACCESS SKILLS for Number and Quantity Vector and Matrix Quantities Standards in High School

Less Complex

More Complex

Standards labeled with a (+) symbol appear in courses intended for all students but are considered “beyond College and Career Ready” and will not be assessed.

Represent and model with vector quantities.	These standards are considered “beyond College and Career Ready” and will not be assessed.
Perform operations on vectors.	These standards are considered “beyond College and Career Ready” and will not be assessed.
Perform operations on matrices and use matrices in applications.	These standards are considered “beyond College and Career Ready” and will not be assessed.

High School Conceptual Category – Algebra

	Standards	Entry Points	Access Skills
Seeing Structure in Expressions	Page 184	Pages 185 – 186	Pages 185 – 187
Arithmetic with Polynomials and Rational Expressions	Page 188	Page 189	Pages 189 – 190
Creating Equations	Page 192	Pages 193 – 194	Pages 193 – 194
Reasoning with Equations and Inequalities	Pages 195 – 196	Pages 197 – 198	Pages 197 – 199

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Seeing Structure in Expressions

High School

Cluster	Standards as written	
Interpret the structure of exponential, polynomial, and rational expressions.	H.A-SSE.1	Interpret expressions that represent a quantity in terms of its context.*
	H.A-SSE.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
	H.A-SSE.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.</i>
	H.A-SSE.2	Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i>
Write expressions in equivalent forms to solve problems.	H.A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	H.A-SSE.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
	H.A-SSE.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	H.A-SSE.3c	Use the properties of exponents to transform expressions for exponential functions. <i>For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i>
	H.A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. * <i>For example, calculate mortgage payments.</i>

* indicates Modeling standard

ENTRY POINTS and ACCESS SKILLS for Algebra

Seeing Structure in Expressions Standards in High School

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Interpret the structure of exponential, polynomial, and rational expressions.	<ul style="list-style-type: none"> Respond to materials used to add or subtract known and unknown quantities (e.g., 3 plus how many equals 5?) Track materials used to add or subtract known and unknown quantities Grasp materials used to add or subtract known and unknown quantities Use two hands to hold materials used to add or subtract known and unknown quantities Release materials used to add or subtract known and unknown quantities Move materials used to add or subtract known and unknown quantities Orient materials used to add or subtract known and unknown quantities Manipulate objects used to add or subtract known and unknown quantities 	<ul style="list-style-type: none"> Identify parts of an expression (e.g., addend, factor, base, exponent, coefficient, constant, etc.) Describe addition and subtraction expressions in math terms or create such expressions from a description (e.g., $x - 4$ represents four less than a number, and six more than a number is represented by $n + 6$) Create a numerical expression involving addition or subtraction that represents a real-life situation using manipulatives, drawings, or technology (e.g., three leaves and four additional leaves is represented by $3 + 4$) Create a numerical expression involving multiplication or division that represents a real-life situation using manipulatives, drawings, or technology (e.g., 5 blocks in each of 3 bins is represented by 3×5) Create a numerical expression involving a mathematical operation 	<ul style="list-style-type: none"> Identify a variable from a context (e.g., four bags, each with the same number of marbles, can be written as $4n$, where n is the number of marbles per bag) Describe a multiplication and division expression in math terms or create such an expression from a description (e.g., $11n$ represents eleven times a number, and twelve divided by a number is represented by $\frac{12}{x}$) Create an addition and subtraction expression from a context (e.g., fourteen people at a party and five more arrive is represented by $14 + 5$, and eating four cookies from a bag of twenty-six cookies is represented by $26 - 4$) Create a mathematical expression by using a minimum of two mathematical terms (e.g., given a coefficient of 12, a variable of n, and a constant of -4, the expression is $12n - 4$) 	<ul style="list-style-type: none"> Determine the greatest common factor (GCF) of an numerical expression (e.g., for 16 and 20, the GCF is 4, and for 23 and 12, it is 1) Determine the greatest common factor of an expression with a variable (e.g., for $6x$ and 10, the GCF is 2, and for $15x$ and $9x$, the GCF is $3x$) Create an expression by using a minimum of four mathematical terms (e.g., given a coefficient 5, a base $2x$, an exponent 3, and a constant 200, the expression is $5(2x)^3 + 200$) Factor an expression by using GCF (e.g., $3n - 39 = 3(n - 13)$ and $4x^2 - 2x = 2x(x - 1)$) Factor an expression using the difference of two squares (e.g., $x^2 - 9 = (x + 3)(x - 3)$) Factor a trinomial expression (e.g., the factored form of $x^2 - 2x - 15$ is $(x - 5)(x + 3)$)

Continue to address skills and concepts that approach grade-level expectations in this cluster

ENTRY POINTS and ACCESS SKILLS for Algebra

Seeing Structure in Expressions Standards in High School

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Write expressions in equivalent forms to solve problems.	<ul style="list-style-type: none"> ◆ Locate objects partially hidden or out of sight needed to add or subtract known and unknown quantities (e.g., remove barrier to expose materials) ◆ Use one object to act on another to add or subtract known and unknown quantities (e.g., use scissors to cut materials to be added or subtracted) ◆ Turn on technology to activate program to add/subtract known and unknown quantities (e.g., turn on technology tool to add/subtract) ◆ Imitate action used to add and subtract known and unknown quantities (e.g., imitate classmate attaching icon to add) 	<ul style="list-style-type: none"> ◆ Demonstrate an understanding of the Associative Property of Addition (e.g., show that $(3 + 12) + 5 = 3 + (12 + 5)$ by equating $15 + 5$ and $3 + 17$) ◆ Demonstrate an understanding of the Associative Property of Multiplication (e.g., show that $9 \cdot (2 \cdot 3) = (9 \cdot 2) \cdot 3$ by equating $9 \cdot 6$ and $18 \cdot 3$) ◆ Demonstrate an understanding of the Commutative Property of Addition by computation (e.g., show that $6 + 5 + 13 = 13 + 6 + 5$) ◆ Demonstrate an understanding of the Commutative Property of Multiplication by computation (e.g., show that $9 \cdot 10 \cdot 2 = 2 \cdot 9 \cdot 10$) 	<ul style="list-style-type: none"> ◆ Demonstrate an understanding of the Distributive Property by computation (e.g., show that $5 \cdot (7 - 2) = 35 - 10$) ◆ Demonstrate an understanding of the Distributive Property using a variable (e.g., show that $5 \cdot (n - 4) = 5n - 20$) ◆ Match expressions to basic properties of operations (e.g., the Commutative Properties of Addition and Multiplication, the Associative Properties of Addition and Multiplication, and the Distributive Property) ◆ Create a division and multiplication expression from a context (e.g., four packages of six muffins is represented by $4 \cdot 6$, and twenty candy bars shared by five people is represented by $\frac{20}{5}$) 	<ul style="list-style-type: none"> ◆ Demonstrate an understanding of the converse of the Distributive Property by computation (e.g., show that $44 + 24 = 4 \cdot (11 + 6)$ and $44 + 24 = 68$ and $4 \cdot 17 = 68$) ◆ Demonstrate an understanding of the converse of the Distributive Property using a variable by factoring (e.g., show that $3n - 15 = 3(n - 5)$) ◆ Use the Distributive Property to evaluate more complex numerical expressions (e.g., $6 \cdot 197 = 6(200 - 3) = 6(200) - 6(3) = 1200 - 18 = 1182$) ◆ Create a numerical expression involving a mathematical operation that represents a real-life situation <p style="text-align: right;"><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ACCESS SKILLS (continued) for Algebra
Seeing Structure in Expressions Standards in High School

Less Complex

More Complex

<u>ACCESS SKILLS</u> <u>The student will:</u>		<u>ENTRY POINTS</u> <u>The student will:</u>
Write expressions in equivalent forms to solve problems. (continued)	<ul style="list-style-type: none"> ♦ Initiate cause-and-effect response during an adding/subtracting activity (e.g., turn on technology tool to activate addition computer program) ♦ Sustain adding/subtracting activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention during adding/subtracting activity (e.g., raise hand vocalize) ♦ Make a request during an activity to add and subtract known and unknown quantities (e.g., request a turn) ♦ Choose from an array of two in an adding and/or subtracting activity. (e.g., choose materials to be used in adding activity) ♦ Attend visually, aurally, or tactilely to materials used to add or subtract known and unknown quantities 	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Arithmetic with Polynomials and Rational Expressions

High School

Cluster	Standards as written	
Perform arithmetic operations on polynomials.	H.A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.
	H.A-APR.1a	Perform operations on polynomial expressions (addition, subtraction, multiplication, division) and compare the system of polynomials to the system of integers when performing operations.
	H.A-APR.1b	Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property
Understand the relationship between zeros and factors of polynomials.	H.A-APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
	H.A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Use polynomial identities to solve problems.	H.A-APR.4	Prove polynomial identities and use them to describe numerical relationships. <i>For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</i>
	H.A-APR.5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
Rewrite rational expressions.	H.A-APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
	H.A-APR.7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

+ indicates standard is beyond College and Career Ready

ENTRY POINTS and ACCESS SKILLS for Algebra Arithmetic with Polynomials and Rational Standards in High School

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u> <u>The student will:</u>	<u>The student will:</u>	<u>ENTRY POINTS</u> <u>The student will:</u>	<u>The student will:</u>
Perform arithmetic operations on polynomials.	<ul style="list-style-type: none">◆ Respond to materials used to add and subtract known and unknown quantities◆ Track materials used to add and subtract known and unknown quantities◆ Shift focus from materials used to add and subtract known and unknown quantities to speaker◆ Grasp materials used to add and subtract known and unknown quantities◆ Use two hands to hold materials used to add and subtract known and unknown quantities◆ Release materials used to add and subtract known and unknown quantities	<ul style="list-style-type: none">◆ Demonstrate the notion of combining like terms in a simple addition problem using visuals or manipulatives (e.g., group bagels, donuts, and muffins without adding them)◆ Classify expressions as polynomials and non-polynomials (e.g., indicate that $4x^2 - 8x$ and $-3t$ are polynomials, while $\frac{3}{x^2}$ is not)◆ Add and subtract two simple binomials by combining like terms (e.g., $(5t + 3) + (t - 3)$)	<ul style="list-style-type: none">◆ Show multiplication of two numerical expressions as the sum of products using the FOIL method (e.g., $5 \cdot 6 = (2 + 3) \cdot (7 - 1) = 2 \cdot 7 + 2 \cdot (-1) + 3 \cdot 7 + 3 \cdot (-1) = 14 - 2 + 21 - 3 = 30$)◆ Add and subtract two binomials with two variables (e.g., $(7x + 3y) - (-2x - 3y)$)◆ Combine like terms for an expression with four or more terms, some of which are like terms (e.g., $4t + 3n - 7t - 6t + 5 - 3n$)	<ul style="list-style-type: none">◆ Add and subtract two or more polynomials, one of which is a trinomial (e.g., $(12x^3 - 14x + 11) - (-2x^2 - 17)$)◆ Multiply two binomials (e.g., $(2x - 3)(2x + 5)$)◆ Solve simple multiplication problems using the zero-product property (e.g., if $n \cdot (n - 6) = 0$, then either $n = 0$ or $n - 6 = 0$, in which case, $n = 6$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

Note: A polynomial is an expression with one or more terms. All variables in a polynomial have a non-negative integer exponent. A monomial is a polynomial with one term. A binomial has two terms and a trinomial has three terms.

Note: FOIL is a method for multiplication of two binomials where each term in one expression is multiplied by each term in the other expression and the products are combined (First, Outer, Inner, Last).

ACCESS SKILLS (continued) for Algebra
Arithmetic with Polynomials and Rational Standards in High School

Less Complex

More Complex

<u>ACCESS SKILLS</u> <u>The student will:</u>		<u>ENTRY POINTS</u> <u>The student will:</u>
Perform arithmetic operations on polynomials. (continued)	<ul style="list-style-type: none"> ◆ Move materials used to add and subtract known and unknown quantities ◆ Orient materials used to add and subtract known and unknown quantities ◆ Manipulate objects used to add and subtract known and unknown quantities ◆ Locate objects partially hidden or out of sight needed to add and subtract known and unknown quantities (e.g., remove barrier to expose materials) ◆ Use one object to act on another to add and subtract known and unknown quantities (e.g., use scissors to cut materials) ◆ Turn on technology used to add and subtract known and unknown quantities (e.g., turn on technology tool to add and subtract) ◆ Imitate action used to add and subtract known and unknown quantities (e.g., imitate classmate attaching icon to add) ◆ Initiate cause-and-effect response during an adding/subtracting activity (e.g., turn on technology tool to activate addition computer program) ◆ Sustain adding/subtracting activity through response (e.g., using preprogrammed voice-generating device comment) ◆ Gain attention during adding/subtracting activity (e.g., raise hand vocalize) ◆ Make a request during an activity to add and subtract known and unknown quantities (e.g., request a turn) ◆ Choose from an array of two in an adding and subtracting activity. (e.g., choose materials to be used in adding activity) ◆ Attend visually, aurally, or tactilely to materials used to add and subtract known and unknown quantities 	

ENTRY POINTS and ACCESS SKILLS for Algebra
Arithmetic with Polynomials and Rational Standards in High School

← Less Complex

More Complex →

Understand the relationship between zeros and factors of polynomials.	These standards are not common to both high school pathways and will not be assessed.
Use polynomial identities to solve problems.	These standards are not common to both high school pathways and will not be assessed.
Rewrite rational expressions.	These standards are not common to both high school pathways and will not be assessed.

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Creating Equations

High School

Cluster	Standards as written	
Create equations that describe numbers or relationships.	H.A-CED.1	Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear and quadratic functions, and simple root and rational functions and exponential functions.) ★
	H.A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
	H.A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i> ★
	H.A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law $V = IR$ to highlight resistance R.</i> ★

ENTRY POINTS and ACCESS SKILLS for Algebra

Creating Equations Standards in High School

← Less Complex

More Complex →

ACCESS SKILLS		ENTRY POINTS		
The student will:		The student will:	The student will:	
Create equations that describe numbers or relationships.	<i>See previous algebra cluster for access skills</i>	<ul style="list-style-type: none">♦ Create a visual model of a one-variable equation using manipulatives or technology (e.g., "I have three pennies and I need eight pennies, how many more pennies do I need?" can be represented by $3 + ? = 8$ using pennies and a non-number as a variable)♦ Create a one-variable equation from a context (e.g., "Maura has two cats" can be represented by $c = 2$)♦ Create a one-variable inequality from a context (e.g., "Jose can watch television for no more than two hours" can be represented by $h \leq 2$)♦ Define the variable in a simple inequality from a context (e.g., if "Tim has more than ten marbles" is represented by $n > 10$, then n = the number of marbles Tim has)	<ul style="list-style-type: none">♦ Create a one-variable equation with an operation from a context (e.g., "A scarf costs twelve dollars and I have 4 dollars; how many more dollars do I need?" can be represented by $12 = 4 + d$)♦ Create a one-variable inequality with an operation from a context (e.g., "I have seven baseball cards and Manuel has 11 baseball cards and I want to have more than him; how many cards should I buy?" can be represented by the inequality $7 + n > 11$)♦ Create a two-variable equation (e.g., "Jake and Maria have a total of 24 mystery books" can be represented by $j + m = 24$)♦ Match the graph of a line on a coordinate plane to the situation it represents (e.g., the graph of a line representing dozens of donuts would be a line through the origin and the points (12,1), (24,2), etc.)	<ul style="list-style-type: none">♦ Create a one-variable equation involving rate from a context (e.g., "if there are 12 cookies for 4 students, how many cookies will each student receive" can be represented by $4n = 12$)♦ Create a one-variable equation involving equivalent rates from a context (e.g., "Sam can run 2 miles in 18 minutes; how many miles can Sam run in 30 minutes?" can be represented by $\frac{18}{2} = \frac{30}{n}$)♦ Create a two-variable equation using multiplication and division from a context (e.g., "There are two boys for every girl" can be represented by $g = 2b$)♦ Create a two-variable inequality using multiplication and division from a context (e.g., "Patty has more than three times as many marbles as Manuel has" can be represented by $p > 3m$)

ENTRY POINTS and ACCESS SKILLS for Algebra

Creating Equations Standards in High School

← Less Complex

More Complex →

<u>ACCESS SKILLS</u>		<u>ENTRY POINTS</u>	
<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Create equations that describe numbers or relationships. (continued)	<ul style="list-style-type: none"> ◆ Define both variables in a two-variable equation given a context (e.g., Nachele and Jon have worked a total of 12 years at the company is represented by $n + j + 12$; n = the number of years Nachele has worked at the company and j = the number of years Jon has worked at the company) ◆ Create an equation by replacing variables in formulas with given values (e.g., if the length of a rectangle is 18 inches and its area is 72 square inches, then, given the formula $A = lw$, $72 = 18w$) 	<ul style="list-style-type: none"> ◆ Identify, given a relationship between two variables, which is dependent, and which is independent (e.g., given distance and time, distance is dependent because distance traveled depends on amount of time spent traveling) 	<ul style="list-style-type: none"> ◆ Graph a line on a coordinate plane that represents a relationship (e.g., one pound of coffee costs \$6.75 and 4 pounds of coffee costs \$27.00; graph the relationship) ◆ Rearrange a literal equation to highlight a different variable (e.g., if $T = \frac{8p}{n}$, then $p = \frac{tn}{8}$; equations can be actual formulas or fabricated) <p style="text-align: right;"><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: This domain is about creating equations and inequalities, not solving them.</p> </div>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Reasoning with Equations and Inequalities

High School

Cluster	Standards as written	
Understand solving equations as a process of reasoning and explain the reasoning.	H.A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.
	H.A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Solve equations and inequalities in one variable.	H.A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	H.A-REI.M A3a	Solve linear equations and inequalities in one variable involving absolute value.
	H.A-REI.4	Solve quadratic equations in one variable.
	H.A-REI.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
	H.A-REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
Solve systems of equations.	H.A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
	H.A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
	H.A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</i>

Solve systems of equations. (continued)	H.A-REI.8	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
	H.A-REI.9	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).
Represent and solve equations and inequalities graphically.	H.A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
	H.A-REI.11	Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
	H.A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

* indicates Modeling standard

+ indicates standard is beyond College and Career Ready

ENTRY POINTS and ACCESS SKILLS for Algebra

Reasoning with Equations and Inequalities Standards in High School

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>
	<u>The student will:</u>	<u>The student will:</u>
Understand solving equations as a process of reasoning and explain the reasoning.	<ul style="list-style-type: none"> Respond to materials used to add and subtract known and unknown quantities Track materials used to add and subtract known and unknown quantities Shift focus from materials used to add and subtract known and unknown quantities 	<ul style="list-style-type: none"> Determine the step needed to solve a one-step equation (e.g., for the equation $3x = 15$, the step would be divide both sides by three)
Solve equations and inequalities in one variable.	<ul style="list-style-type: none"> Determine a first step needed to solve a multi-step equation (e.g., for the equation $5t - 15 = t - 11$, the first step could be to add fifteen to both sides) Explain each step in the solution of an equation (e.g., justify using properties of equality) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>	<ul style="list-style-type: none"> Explain each step in the solution of an equation (e.g., justify using properties of equality)
	<ul style="list-style-type: none"> Determine the step needed to solve a one-step equation (e.g., for the equation $3x = 15$, the step would be divide both sides by three) Determine the absolute value for positive and negative integers using a number line (e.g., -12 is 12 because it is twelve units from 0 on a number line and 9 is 9 because it is nine units from zero on a number line) Solve a one-step equation using addition or subtraction (e.g., $n + 11 = 19$) Solve a one-step equation using multiplication or division (e.g., $4x = 36$) Solve a simple absolute value equation (e.g., $k = 7 + -5$) Simplify an expression using order of operations (e.g., $7 + (5 - 1)^2 \times 2$) 	<ul style="list-style-type: none"> Evaluate a one-variable expression for specific values of the variable (e.g., evaluate the expression $6y - 13$ for $y = 3$ and for $y = -1$) Solve a multi-step equation (e.g., $7x - 10 = 46$) Solve a one-step absolute value equation (e.g., $x + 6 = 14$) Match an inequality with its graph on a number line (e.g., match $n \leq 10$ to its corresponding graph) Solve a more complex equation involving absolute value (e.g., $6 + 2x + 1 = 11$) Determine the inequality from a solution shown on a number line (e.g., for a number line with a ray going to the left, with open endpoint at 7, the inequality is $n < 7$) Graph an inequality on a number line (e.g., graph $t \geq -3$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS and ACCESS SKILLS for Algebra

Reasoning with Equations and Inequalities Standards in High School

← Less Complex

More Complex →

	<u>ACCESS SKILLS</u>	<u>ENTRY POINTS</u>
	<u>The student will:</u>	<u>The student will:</u>
Solve systems of equations.	<ul style="list-style-type: none"> ◆ Manipulate objects used to add and subtract known and unknown quantities ◆ Locate objects partially hidden or out of sight needed to add and subtract known and unknown quantities (e.g., remove barrier to expose materials) ◆ Use one object to act on another to add and subtract known and unknown quantities 	<ul style="list-style-type: none"> ◆ Demonstrate equality properties by scaling or by adding to both sides of a numerical equation (e.g., for the equation $6 + 14 = 20$, multiply both sides by 2 to reveal a true equation) ◆ Determine the point of intersection of two lines graphed on a coordinate plane by inspection (e.g., the point of intersection of two lines is $(5, -1)$)
Represent and solve equations and inequalities graphically.	<ul style="list-style-type: none"> ◆ Turn on technology used to add and subtract known and unknown quantities (e.g., turn on technology tool to add and subtract) ◆ Gain attention during adding/subtracting activity (e.g., raise hand vocalize) ◆ Attend visually, aurally, or tactilely to materials used to add and subtract known and unknown quantities 	<ul style="list-style-type: none"> ◆ Multiply both sides of an equation by an integer (e.g., given the equation $2x - 3y = 16$, multiply both sides by 2 to get $4x - 6y = 32$) ◆ Determine the point of intersection of two lines graphed on a coordinate plane by inspection (e.g., the point of intersection of two lines is $(5, -1)$) ◆ Determine the point of intersection of two lines graphed on a coordinate plane by inspection (e.g., the point of intersection of two lines is $(5, -1)$)
		<ul style="list-style-type: none"> ◆ Determine the point of intersection of two lines graphed on a coordinate plane by solving algebraically (e.g., using substitution or elimination) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
		<ul style="list-style-type: none"> ◆ Determine whether an ordered pair satisfies an inequality in two variables (e.g., by inspecting a graph) ◆ Graph a simple inequality on a coordinate grid (e.g., graph $x > 1$ or $y \leq x - 3$) ◆ Determine whether an ordered pair satisfies a system of inequalities (e.g., by inspecting a graph) ◆ Graph a system of inequalities on a coordinate grid (e.g., graph $y > x - 1$ and $y \leq 2x + 3$) ◆ Determine algebraically whether an ordered pair satisfies a system of inequalities (e.g., by substituting the coordinates into each inequality) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ACCESS SKILLS (continued) for Algebra
Reasoning with Equations and Inequalities Standards in High School

Less Complex

More Complex

<u>ACCESS SKILLS</u> <u>The student will:</u>		<u>ENTRY POINTS</u> <u>The student will:</u>
Represent and solve equations and inequalities graphically. (continued)	<ul style="list-style-type: none"> ◆ Imitate action used to add and subtract known and unknown quantities (e.g., imitate classmate attaching icon to add) ◆ Initiate cause-and-effect response during an adding/subtracting activity (e.g., turn on technology tool to activate addition computer program) ◆ Sustain adding/subtracting activity through response (e.g., using preprogrammed voice-generating device comment) ◆ Make a request during an activity to add and subtract known and unknown quantities (e.g., request a turn) ◆ Choose from an array of two in an adding and subtracting activity (e.g., choose materials to be used in adding activity) 	

High School Conceptual Category – Functions

	Standards	Entry Points	Access Skills
Interpreting Functions	Pages 201 – 202	Pages 203 – 205	
Building Functions	Pages 206 – 207	Page 208	
Linear, Quadratic, and Exponential Models	Page 209	Pages 210 – 211	
Trigonometric Functions	Page 212	Page 213	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Interpreting Functions

High School

Cluster	Standards as written	
Understand the concept of a function and use function notation.	H.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
	H.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. <i>For example, given a function representing a car loan, determine the balance of the loan at different points in time.</i>
	H.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.</i>
Interpret functions that arise in applications in terms of the context (linear, quadratic, exponential, rational, polynomial, square root, cube root, trigonometric, logarithmic).	H.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
	H.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★
	H.F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
Analyze functions using different representations.	H.F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
	H.F-IF.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
	H.F-IF.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★

★ indicates Modeling standard

H.F- IF.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. *
H.F- IF.7d	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. *
H.F- IF.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. *
H.F- IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
H.F- IF.8a	Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
H.F- IF.8b	Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase. <i>For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</i>
H.F- IF.9	Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. <i>For example, given a graph of one polynomial function (including quadratic functions) and an algebraic expression for another, say which has the larger/smaller relative maximum and/or minimum.</i>
H.F- IF.MA 10	Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

* indicates Modeling standard

+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions

Interpreting Functions Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand the concept of a function and use function notation.	<ul style="list-style-type: none"> ◆ Complete an input-output table when given the function rule and some values (e.g., student fills in missing values in a table) ◆ Identify the quantitative dependent and independent variable in a real-life situation (e.g., the number of students going on a trip (independent) and the number of buses required (dependent)) ◆ Determine whether a relationship illustrated by a set of ordered pairs or a table, involving a domain (input) and range (output) represents a function (e.g., is the relationship represented by the ordered pairs (3,4) (5,12) (3,8) a function?) ◆ Evaluate a function for a value of its domain (e.g., for the function $f(x) = -2x$ find $f(8)$) ◆ Determine the initial value (where $x = 0$) of a function in the form $f(x) = mx + b$ (e.g., identify the y-intercept of the function) ◆ Determine the rate of change of a function from its graph (e.g., by calculating slope) ◆ Determine the rate of change of a function from a table of ordered pairs (e.g., by calculating rate of change between two ordered pairs) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Determine whether the relationship shown in an a list of ordered pairs, an input-output table, or a mapping represents a function (e.g., using a variety of representations) ◆ Evaluate a function for multiple values of its domain (e.g., for the function $f(x) = 2x + 1$, find $f(-1)$, $f(4)$, $f(5)$, etc.) ◆ Interpret functional relationships in terms of a context (e.g., from a table, the graph of a line, a description, etc.) 	<ul style="list-style-type: none"> ◆ Determine whether the relationship shown in a graph represents a function (e.g., by using the vertical line test) ◆ Determine whether the relationship shown in a variety of ways represents a function (e.g., table, mapping, graph, etc.) ◆ Solve a problem involving dependent and independent variables in a real-life situation (e.g., determine the number of buses needed given the number of students going on a trip) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Functions

Interpreting Functions Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Interpret functions that arise in applications in terms of the context (linear, quadratic, exponential, rational, polynomial, square root, cube root, trigonometric, logarithmic).	<ul style="list-style-type: none"> ◆ Interpret positive or negative rate of change of a linear function in terms of a context (e.g., a negative rate of change in a distance/time graph indicates deceleration) ◆ Interpret mathematical relationships in terms of a context (e.g., from a table, graph, description, etc.) ◆ Create a table of ordered pairs that represents a relationship between two variables in a real-life situation (e.g., \$0.95 per donut) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Interpret the rate of change of a linear function in terms of a context (e.g., in a function that represents miles traveled over time, the slope represents the average speed in miles per hour) ◆ Interpret the initial value (y-intercept) of a linear function in terms of a context (e.g., the y-intercept of a graph showing profits at a car wash represents the amount of money initially spent) ◆ Create a graph that represents the relationship between two variables in a real-life situation (e.g., the total cost of sodas at \$0.85 each) 	<ul style="list-style-type: none"> ◆ Identify the set of numbers to include as possible input values (domain) in a real-life situation (e.g., if ribbon costs a dollar per yard, the domain for the total yardage of ribbon is the set of non-negative real numbers) ◆ Identify the set of numbers to include as possible output values (range) in a real-life problem that includes real numbers (e.g., for buying eggs by the dozen, the range of values for the total number of eggs is positive multiples of 12) ◆ Interpret the rate of change over an interval of the domain on a graph of a non-linear function (e.g., for a distance/time piecewise graph determine an average rate of time between two points) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Functions

Interpreting Functions Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Analyze functions using different representations. (F-IF)	<ul style="list-style-type: none"> ◆ Graph a linear function on a coordinate grid given a table of ordered pairs (e.g., graph the line that passes through two points) ◆ Describe a graph of a (piecewise) function that has labeled sections (e.g., part A rises gently, part B is short and flat, part C rises like part A, but is steeper, etc.) ◆ Match a graph of a function with its equation (e.g., using linear functions with different features) ◆ Match an equation of a function with a table of ordered pairs (e.g., connect the equation with the values that satisfy it) ◆ Match ordered pairs with its graph (e.g., list coordinate points or a table with ordered pairs) ◆ Identify graphs of linear, exponential, and quadratic functions (e.g., differentiate between them) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Graph an exponential function on a coordinate grid given a table of ordered pairs (e.g., roughly show any intercepts, asymptote, and end behavior) ◆ Graph a quadratic function on a coordinate grid given a table of ordered pairs (e.g., show any intercepts, the vertex, and end behavior) ◆ Create a graph of a (piecewise) function based on a description (e.g., describe graphically distance traveled on a bike ride over time.) ◆ Create a table of ordered pairs based on a quadratic function (e.g., create a table for $f(x) = x^2 - 3$) ◆ Create a table of ordered pairs based on an exponential function (e.g., create a table for $f(x) = 2^x$) ◆ Compare initial values of two functions presented in different ways (e.g., for $f(x) = 3x - 5$ the initial value is -5, and for the relationship (0,2), (1,0), (2,-2), the initial value is 2) 	<ul style="list-style-type: none"> ◆ Graph a linear, exponential, or quadratic function on a coordinate grid given its equation (e.g., graph $f(x) = -2x - 8$, $g(x) = 3^x$, etc.) ◆ Compare the rates of change for two functions presented in different ways (e.g., compare the rates of change of Ella's savings account, based on a description, and Kevin's savings account, based on an equation) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Building Functions

High School

Cluster	Standards as written	
Build a function that models a relationship between two quantities.	H.F-BF.1	Write a function (linear, quadratic, exponential, simple rational, radical, logarithmic, and trigonometric) that describes a relationship between two quantities. ★
	H.F-BF.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
	H.F-BF.1b	Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. ★</i>
	H.F-BF.1c	(+) Compose functions. <i>For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time. ★</i>
	H.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★
Build new functions from existing functions.	H.F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.)
	H.F-BF.4	Find inverse functions algebraically and graphically.
	H.F-BF.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. (Include linear and simple polynomial, rational, and exponential functions.)
	H.F-BF.4b	(+) Verify by composition that one function is the inverse of another.
	H.F-BF.4c	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
	H.F-BF.4d	(+) Produce an invertible function from a non-invertible function by restricting the domain.
	H.F-BF.5	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

★ indicates Modeling standard + indicates standard is beyond College and Career Ready

+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions

Building Functions Standards in High School

Less Complex

More Complex

<p>Build a function that models a relationship between two quantities.</p>	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Determine the missing value in a table of ordered pairs that represents a linear function (e.g., a missing output value) ◆ Determine the missing value within a given arithmetic sequence by identifying an initial value and an addition/subtraction rule (e.g., 12, 7, 2, ?, -8 ...) ◆ Complete a table to extend an arithmetic sequence (e.g., complete a partial table that has some values and a linear function rule) ◆ Create an arithmetic sequence using manipulatives (e.g., add a constant number of objects) <p><i>See entry points for earlier grades in this domain</i></p>	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Determine the rule that defines a function based on a table of ordered pairs (e.g., the rule for values in a table with inputs 1, 2, 3, and corresponding outputs 11, 16, 21, is $5x + 6$) ◆ Determine the missing value within a given geometric sequence by identifying an multiplication/division rule (e.g., 2, 6, ?, 54, 162 ...) ◆ Determine a specific term outside of a given arithmetic sequence (e.g., determine the 12th term in the sequence) 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Create a sequence of ordered pairs in a table based on a function rule (e.g., for $f(x) = -3x - 11$ find $f(1)$, $f(2)$, $f(3)$. etc.) ◆ Determine a specific term outside of a given geometric sequence (e.g., determine the 8th term in the sequence) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
<p>Build new functions from existing functions.</p>	<ul style="list-style-type: none"> ◆ Identify the graph of a parent function (e.g., $y = x$, $y = x^2$, $y = x$, etc.) ◆ Draw the graph of a parent function (e.g., make a rough sketch of $y = x^2$) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Match the graph of a function to a graph of its inverse (e.g., choose the graph that demonstrates symmetry over the line $y = x$) ◆ Graph a function over a translation of a parent graph (e.g., for $y = x$, graph $y = x - 3$) 	<ul style="list-style-type: none"> ◆ Create the equation of a translation of a parent function based on its graph (e.g., The translation of 2 units to the left of a quadratic function has the equation $y = (x + 2)^2$) ◆ Calculate the equation of the inverse of a linear function (e.g., for an equation in the form $y = mx + b$, use the equation $x = my + b$ and solve for y.) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Linear, Quadratic, and
Exponential Models

High School

Cluster	Standards as written	
Construct and compare linear, quadratic, and exponential models and solve problems.	H.F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. *
	H.F-LE.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. *
	H.F-LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. *
	H.F-LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. *
	H.F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). *
	H.F-LE.3	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *
	H.F-LE.4	For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology. *
Interpret expressions for functions in terms of the situation they model.	H.F-LE.5	Interpret the parameters in a linear or exponential function (of the form $f(x) = bx + k$) in terms of a context. *

* indicates Modeling standard

ENTRY POINTS for Functions

Linear, Quadratic, and Exponential Models Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Construct and compare linear, quadratic, and exponential models and solve problems.	<ul style="list-style-type: none"> ◆ Create an arithmetic sequence (e.g., a sequence of numbers with a common difference) ◆ Identify a real-life situation that can be modeled by a linear function from a table, graph, description, or symbols (e.g., the cost of hamburgers at \$4 each) ◆ Construct a linear function that represents the values of ordered pairs shown in a table (e.g., for x values of 3, 4, and corresponding $f(x)$ values of 7, 10, the function is $f(x) = 3x - 2$) ◆ Distinguish between linear and non-linear models based on tables representing them (e.g., a table that does not show a constant increase represents a non-linear relationship) ◆ Distinguish between situations that are modeled by exponential growth and by exponential decay (population increase vs. a car's depreciation) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Create a geometric sequence (e.g., by identifying an initial value and a multiplication/division rule) ◆ Identify a real-life situation that can be modeled by an exponential function from a table, graph, description, or symbol (e.g., the sale price of an item reduced by 10% per week) ◆ Construct an exponential function that represents the values of ordered pairs shown in a table (e.g., for x values of 1, 3, and corresponding $f(x)$ values of 2, 18, the function is $f(x) = \frac{2}{3}(3^x)$) ◆ Interpret a table that represents a real-life situation or mathematical relationship that can be modeled by a linear function, from a table, graph, description, or symbols (e.g., by finding any rate of increase/decrease and initial value) ◆ Distinguish between linear, exponential, and quadratic models (e.g., from a table, equation, graph, description, etc.) 	<ul style="list-style-type: none"> ◆ Distinguish between a linear model and a non-linear model based on a real-life situation (e.g., bank account growth from deposits vs. growth from interest) ◆ Distinguish between exponential and non-exponential models based on tables representing them (e.g., a table that does not show a constant increase represents a non-linear relationship) ◆ Interpret a table that represents a real-life situation or mathematical relationship that can be modeled by an exponential function, from a table, graph, description, or symbol (e.g., by finding any growth/decay rate and initial value) ◆ Compare a linear, an exponential, and a quadratic function for several inputs (e.g., to illustrate that exponential growth will eventually exceed linear and quadratic growth) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Functions

Linear, Quadratic, and Exponential Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Interpret expressions for functions in terms of the situation they model. (continued)	<ul style="list-style-type: none"> Identify the parameters of a linear function (e.g., for the function $f(x) = mx + b$, m represents the rate of change and b represents the initial value) Identify the parameters of an exponential function (e.g., for the function $f(x) = a(b)^x$, a represents the initial value, b represents the growth/decay factor, and x represents the independent variable) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Identify the parameters of a linear function that models a real-life situation (e.g., $f(x) = 8.5x - 25$ represents profits from a car wash; \$8.50 represents the amount charged per car wash and \$25 represents the cost of materials) Compare the parameters of 2 or more linear functions from a table or a description (e.g., which has the greater rate of change and/or initial value) Compare the parameters of 2 or more linear functions that represent real-life situations from a table or a description (e.g., which has the greater rate of change and/or initial value) 	<ul style="list-style-type: none"> Identify the parameters of an exponential function that models a real-life situation (e.g., $f(x) = 100(1.02)^6$ represents Leo's bank account; \$100 is the initial value, 2% is the growth rate, 6 is the number of years) Compare the parameters of 2 or more exponential functions from a table or a description (e.g., which has the greater growth/decay rate and/or initial value) Compare the parameters of 2 or more exponential functions that represent real-life situations from a table or a description (e.g., which has the greater growth/decay rate and/or initial value) Compare the parameters of 2 or more functions of different types that represent real-life situations from a table or a description (e.g., which has the greater initial value and/or which has the greater value over an interval of the independent variable between a linear and an exponential model) <p style="text-align: right;"><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Trigonometric Functions

High School

Cluster	Standards as written	
Extend the domain of trigonometric functions using the unit circle.	H.F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
	H.F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
	H.F-TF.3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
	H.F-TF.4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions.	H.F-TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. *
	H.F-TF.6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
	H.F-TF.7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. *
Prove and apply trigonometric identities.	H.F-TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.
	H.F-TF.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

* indicates Modeling standard

+ indicates standard is beyond College and Career Ready

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

MATHEMATICS

ENTRY POINTS for Functions

Trigonometric Functions Standards in High School

Less Complex

More Complex

The student will:

The student will:

The student will:

Extend the domain of trigonometric functions using the unit circle.

These standards are not common to both high school pathways and will not be assessed.

Model periodic phenomena with trigonometric functions.

These standards are not common to both high school pathways and will not be assessed.

Prove and apply trigonometric identities.

These standards are not common to both high school pathways and will not be assessed.

High School Conceptual Category – Geometry

	Standards	Entry Points	Access Skills
Congruence	Pages 215 – 216	Pages 217 – 219	
Similarity, Right Triangles, and Trigonometry	Pages 220 – 221	Pages 222 – 223	
Circles	Page 224	Page 225	
Expressing Geometric Properties with Equations	Page 226	Page 227	
Geometric Measurement and Dimension	Page 228	Page 229	
Modeling with Geometry	Page 230	Page 231	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Congruence

High School

Cluster	Standards as written	
Experiment with transformations in the plane.	H.G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
	H.G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
	H.G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
	H.G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
	H.G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Understand congruence in terms of rigid motions.	H.G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
	H.G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
	H.G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Prove geometric theorems and, when appropriate, the converse of theorems.	H.G-CO.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
	H.G-CO.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.
	H.G-CO.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

	H.G- CO.MA 11a	Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.
Make geometric constructions.	H.G- CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
	H.G- CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

ENTRY POINTS for Geometry

Congruence Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Experiment with transformations in the plane.	<ul style="list-style-type: none"> Match the names of geometrical figures to their drawings (e.g., angle, circle, perpendicular lines, parallel lines, line segment, etc.) Demonstrate a translation of a two-dimensional figure using manipulatives (e.g., slide a figure on a calibrated table or graph paper) Demonstrate a reflection of a two-dimensional figure using manipulatives (e.g., flip a figure on a calibrated table or graph paper) Demonstrate a rotation of a two-dimensional figure using manipulatives (e.g., turn a figure on a calibrated table or graph paper) Show congruence of two figures (e.g., using technology or by tracing/placing a transparency over two figures to show that they are the same) On a coordinate plane translate a given figure a number of units vertically or horizontally (e.g., translate a triangle 4 units left) Distinguish between a translation, a reflection, and a rotation (e.g., from a description) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Match geometrical figures to their definitions (e.g., angle, circle, perpendicular lines, parallel lines, line segment, etc.) Demonstrate a translation reflection and/or rotation of a two-dimensional figure on a coordinate grid (e.g., using technology or by drawing on a coordinate grid) Identify a translation, reflection, and/or rotation (e.g., given a pre-image and an image on a coordinate grid, determine the type of transformation that produced the image) Rotate a figure on a coordinate plane (e.g., rotate a triangle 90° clockwise, 180° counterclockwise, etc.) Identify types of symmetry within a figure (e.g., the figure has both reflection symmetry and rotational symmetry) Determine whether figures are congruent based on a series of transformations (e.g., using a combination of translations, reflections, rotations, and dilations) Determine the coordinates of a point after a transformation (e.g., after a reflection over the x-axis, the point $(5, -9)$ maps to $(5, 9)$) Use a transformation or a sequence of transformations to map a figure onto itself (e.g., a reflection over the y-axis, a reflection over the x-axis, and then a rotation 180° about the origin) 	<ul style="list-style-type: none"> Identify geometrical figures based on their description (e.g., "the set of points equidistant from a given point in a plane" is a circle) Draw any lines of symmetry exhibited by a two-dimensional figure (e.g., using a ruler draw the four line of symmetry exhibited in a square) Determine the degrees of rotational symmetry exhibited by a figure (e.g., an equilateral triangle exhibits 120° and 240° rotational symmetry) Determine the coordinates of the vertices of a figure on a coordinate grid after a transformation (e.g., translate a triangle 3 units left and list the coordinates of the image) Determine the coordinates of the vertices of a figure on a coordinate grid after a series of transformations (e.g., reflect a triangle over the y-axis and then translate it 2 units down and list the coordinates of the image) Determine whether a transformation or a sequence of transformations maps a figure onto itself (e.g., a reflection over the y-axis and then a rotation 90° clockwise about the origin does not map it to itself) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Geometry

Congruence Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand congruence in terms of rigid motions.	<ul style="list-style-type: none"> Identify shapes that are congruent* using SSS (e.g., given triangles that include side lengths, determine which are congruent) Identify shapes that are congruent using SAS (e.g., given triangles that include side lengths and an angle measure, determine which are congruent) Identify shapes that are congruent using ASA (e.g., given triangles that include angle measures and one side length, determine which are congruent) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Determine a missing side length, given congruent triangles (e.g., given a triangle with side lengths 5 and 6 units and an included angle measure of 71°, and a triangle congruent by SAS with side length of 5 units and angle measure 71°, show which side has a length of 6 units) Decide which postulate (i.e., SSS, SAS, or ASA) can be used to prove two (given) triangles are congruent <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>* Congruent: A figure is <i>congruent</i> to another figure when both have the same size and shape.</p> </div>	<ul style="list-style-type: none"> Determine whether triangles are congruent based on side lengths or angle measures (e.g., given a triangle, choose another that must be congruent) Determine a missing side length or angle measure given congruent triangles (e.g., given triangles congruent by ASA, find a missing angle measure) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Prove geometric theorems and, when appropriate, the converse of theorems.	<ul style="list-style-type: none"> Identify vertical angles (e.g., in diagram with intersecting lines and/or segments) Identify corresponding angles (e.g., in a diagram with intersecting lines and one or more transversals) Identify alternate interior angles (e.g., in a diagram with intersecting lines and one or more transversals) Use properties of isosceles triangles to solve for missing angle measures (e.g., given an isosceles triangle and an angle measure, determine the measures of the other angles in the triangle) Determine whether a line bisects a line segment (e.g., with markings or by measuring) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Determine the measure of an angle using the Vertical Angle Theorem (e.g., in a simple or complex diagram) Determine the measure of an angle using the Corresponding Angles Postulate (e.g., in a simple or complex diagram) Determine the measure of an angle using the Alternate Interior Angles Theorem (e.g., in a simple or complex diagram) Determine missing angle measures or side length in parallelograms (based on properties of parallelograms) 	<ul style="list-style-type: none"> Determine missing angle measures (e.g., in a simple or complex diagram) using postulates or theorems Given angle measures and side lengths, determine if a figure is a parallelogram (based on properties of parallelograms) Given a parallelogram, determine missing side lengths or angle measures (based on properties of parallelograms) Determine the measure of angles in a regular polygon (e.g., by dividing the polygon into triangles) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Geometry

Congruence Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Make geometric constructions.	<p><i>Using a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.):</i></p> <ul style="list-style-type: none"> ◆ Identify figures and parts of figures by their geometric name (e.g., pentagon, line segment, semicircle etc.) ◆ Draw two-dimensional geometric figures (e.g., using a compass or straightedge) ◆ Draw an angle of a given measure (e.g., using a protractor) <p><i>See entry points for earlier grades in this domain</i></p>	<p><i>Using a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.):</i></p> <ul style="list-style-type: none"> ◆ Construct a perpendicular line (e.g., with a compass and straightedge) ◆ Construct a perpendicular bisector (e.g., with a compass and straightedge) 	<p><i>Using a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.):</i></p> <ul style="list-style-type: none"> ◆ Construct parallel lines (e.g., with a compass and straightedge) ◆ Construct an angle bisector (e.g., with a compass and straightedge) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Similarity, Right Triangles,
and Trigonometry

High School

Cluster	Standards as written	
Understand similarity in terms of similarity transformations.	H.G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: (1a, 1b)
	H.G-SRT.1a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	H.G-SRT.1b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
	H.G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
	H.G-SRT.3	Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.
Prove theorems involving similarity.	H.G-SRT.4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
	H.G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Define trigonometric ratios and solve problems involving right triangles.	H.G-SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
	H.G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.
	H.G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

* indicates Modeling standard

Apply trigonometry to general triangles.	H.G-SRT.9	(+) Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	H.G-SRT.10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.
	H.G-SRT.11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Geometry

Similarity, Right Triangles, and Trigonometry Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand similarity in terms of similarity transformations.	<ul style="list-style-type: none"> Determine if figures appear similar* (e.g., choose a figure similar to a given figure among figures that are not similar) Demonstrate a dilation of a two-dimensional figure using manipulatives or technology (e.g., on a drawing program) Match similar figures (e.g., with proportional triangles and other plane figures) Determine a missing angle measure in a triangle given the other measures (e.g., given a triangle with angle measures of 67° and 59° find the other angle measure) <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>* Similar: Figures with the same angle measures and proportional side lengths are <i>similar</i>.</p> </div> <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Given a preimage on a coordinate grid, a center of dilation, and a scale factor, draw the resulting figure (e.g., dilate a triangle using the origin as the center of dilation with a scale factor 3) Given the lengths of the sides of a figure and a scale factor of dilation, determine the lengths of the sides of the image (a side length of 4 units becomes 2 units with a scale factor of $\frac{1}{2}$) Given two of the angle measures of two triangles, determine if they are similar (e.g., triangle A has angle measures of 29° and 98° and triangle B has angle measures of 29° and 53°, are they similar?) 	<ul style="list-style-type: none"> Demonstrate a sequence of transformations that exhibits the similarity between two given figures on a coordinate grid (e.g., rotate 90° clockwise, translate 2 units left, and the dilate by scale factor of 2 with respect to the origin) Find the scale factor given a dilated figure and its preimage and some of their measurements (e.g., on a coordinate grid) Determine the center of dilation for two similar figures (e.g., by extending their sides) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Prove theorems involving similarity.	<ul style="list-style-type: none"> Calculate the side lengths of a figure, similar to another, given dimensions and a scale factor (e.g., given a triangle with sides 5, 6, and 10 and a scale factor 2, the side lengths of the image are 10, 12, and 20) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Find the missing angle measure in a similar triangles diagram (e.g., using the AA similarity postulate) Find the missing side length in a similar triangles diagram (e.g., using a proportional relationship) 	<ul style="list-style-type: none"> Solve problems with similar figures (e.g., find a missing angle measure and/or a missing side length) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Geometry

Similarity, Right Triangles, and Trigonometry Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Define trigonometric ratios and solve problems involving right triangles.	<ul style="list-style-type: none"> Identify and label the components of a right triangle (e.g., hypotenuse, legs, right angle) Distinguish between different types of triangles (e.g., isosceles, right, equilateral, obtuse) Identify special right triangles (i.e., 30°-60°-90°; 45°-45°-90°) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Use the Pythagorean Theorem to find the length of the hypotenuse or sides of a right triangle (e.g., find the length of the hypotenuse of a right triangle with leg lengths of 6 and 10 units) Calculate trigonometric values in right triangles (e.g., calculate the value of the tangent of an angle with adjacent side length of 7 units and opposite side length of 9 units) 	<ul style="list-style-type: none"> Apply the Pythagorean Theorem in a real-life situation to find a missing length (e.g., the length of a ladder leaning against a wall) Use trigonometry to solve side lengths and angle measures in right triangles (e.g., angle of elevation problems) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Apply trigonometry to general triangles.	These standards are considered "beyond College and Career Ready" and will not be assessed.		

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Circles

High School

Cluster	Standards as written	
Understand and apply theorems about circles.	H.G-C.1	Prove that all circles are similar.
	H.G-C.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
	H.G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.
	H.G-C.4	(+) Construct a tangent line from a point outside a given circle to the circle.
Find arc lengths and areas of sectors of circles.	H.G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

+ indicates standard is beyond College and Career Ready

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

MATHEMATICS

ENTRY POINTS for Geometry Circles Standards in High School

← Less Complex

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand and apply theorems about circles.	<ul style="list-style-type: none"> ◆ Create circles with a given radius using a compass (e.g., draw a circle with a radius of 4 inches) ◆ Define vocabulary associated with circles (e.g., by matching, multiple choice, etc.) ◆ Draw inscribed and central angles in a circle (e.g., inscribe a 40° angle) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Demonstrate the relationship between inscribed and central angles (calculate the measure of the inscribed angle subtended by a 72° central angle) ◆ Solve problems involving the relationship between radius, diameter, and circumference of circles (e.g., calculate area given diameter, calculate radius given circumference, etc.) ◆ Determine the sum of the measures of interior angles of a given polygon (e.g., using the formula $180(n - 2)^\circ$) 	<ul style="list-style-type: none"> ◆ Solve problems involving the relationship between inscribed and central angles (e.g., find a missing angle measure in a diagram) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Find arc lengths and areas of sectors of circles.	<ul style="list-style-type: none"> ◆ Calculate the area of a circle or a semicircle (a semicircle has a radius of 5 units; what is its area?) ◆ Determine the circumference of a circle given its diameter or radius (e.g., using π) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Use the relationship between an arc and an angle in a circle to solve problems (find the measure of a major arc based on a measure of an inscribed angle) ◆ Find arc measures, in degrees, of circles by applying concept of central and inscribed angles (e.g., find the measure of an arc subtended by a 146° inscribed angle) 	<ul style="list-style-type: none"> ◆ Find arc lengths, in radians, of circles by applying concepts of radius, diameter, and circumference (e.g., using the unit circle) ◆ Find arc measures, in degrees, of circles by applying concept of central and inscribed angles (e.g., find the measure of an arc subtended by a 146° inscribed angle) ◆ Find the area of a sector of a circle (find the area of the sector within a 50° central angle of a circle with a radius of 12 units) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Expressing Geometric Properties
 with Equations

High School

Cluster	Standards as written	
Translate between the geometric description and the equation for a conic section.	H.G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
	H.G-GPE.2	Derive the equation of a parabola given a focus and directrix.
	H.G-GPE.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
	H.G-GPE.MA3a	(+) Use equations and graphs of conic sections to model real-world problems. *
Use coordinates to prove simple geometric theorems algebraically.	H.G-GPE.4	Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.</i>
	H.G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
	H.G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
	H.G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula). *

* indicates Modeling standard

+ indicates standard is beyond College and Career Ready

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

ENTRY POINTS for Geometry

Expressing Geometric Properties with Equations Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Translate between the geometric description and the equation for a conic section.	<ul style="list-style-type: none"> Draw a circle with a given radius and center at the origin on a coordinate grid (e.g., with a compass) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Draw a circle, with center at the origin, given an equation (e.g., graph $x^2 + y^2 = 9$) Draw a circle given a radius and center on a coordinate grid (draw a circle with radius 4 and center $(-1,3)$) Determine the equation of a circle, with a center at the origin on the coordinate plane, given the radius (e.g., using $x^2 + y^2 = r^2$) 	<ul style="list-style-type: none"> Graph a circle given an equation in the form $(x - h)^2 + (y - k)^2 = r^2$ (e.g., with center (h,k) and radius r) Determine the equation of a circle based on its graph (e.g., a circle with center $(1,-2)$ and radius 12 has the equation $(x - 1)^2 + (y + 2)^2 = 144$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Use coordinates to prove simple geometric theorems algebraically.	<ul style="list-style-type: none"> Determine the distance between two points on a horizontal or vertical number line (e.g., based on the line's calibration) Identify slope as positive, negative, zero, or undefined Create a figure by graphing the coordinates of its vertices (e.g., graph and connect four ordered pairs to create a quadrilateral) Determine the midpoint of two given points on a number line (e.g., find the midpoint between -5 and 9) Illustrate concepts of parallel, perpendicular, midpoint, and slope of a line (e.g., draw, identify, label etc.) Determine the length of horizontal or vertical line segments on a coordinate plane (e.g., find the length of a segment with endpoints $(3,4)$ and $(-8,4)$) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> Determine the coordinates of the midpoint of a vertical or horizontal line segment on the coordinate plane (e.g., find the midpoint between $(5,7)$ and $(5,-2)$) Determine the slope of a line on a coordinate grid Determine the length of a line segment on a coordinate plane (e.g., using the Pythagorean Theorem) Determine the midpoint of a line segment on a coordinate grid (e.g., find the midpoint of a segment with endpoints $(1,0)$ and $(7,-8)$) Determine the length of a line segment on a coordinate grid (e.g., using the Pythagorean Theorem, distance formula, etc.) 	<ul style="list-style-type: none"> Determine the perimeter of a polygon on a coordinate grid (e.g., using the Pythagorean Theorem, distance formula) Determine the area of a polygon on a coordinate plane (e.g., using the Pythagorean Theorem, distance formula, etc.) Determine the slope of a line parallel or perpendicular to a given line (e.g., what is the slope of a line parallel to the line with equation $y = 3x - 1$?) Determine the slope of two lines on a coordinate grid to determine if they are parallel or perpendicular on a coordinate plane (e.g., lines may appear to be but may not be parallel/perpendicular) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Geometric Measurement
and Dimension

High School

Cluster	Standards as written	
Explain volume formulas and use them to solve problems.	H.G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i>
	H.G-GMD.2	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
	H.G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *
Visualize relationships between two-dimensional and three-dimensional objects.	H.G-GMD.4	Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

★ indicates Modeling standard

+ indicates standard is beyond College and Career Ready

Alternate Academic Achievement Standards for Massachusetts Curriculum Frameworks (Resource Guide) Fall 2023

MATHEMATICS

ENTRY POINTS for Geometry

Geometric Measurement and Dimension Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Explain volume formulas and use them to solve problems.	<ul style="list-style-type: none"> ◆ Match volume formulas to objects as a means of identifying the object (e.g., match a cone to its volume formula) ◆ Calculate the area of a plane figure (e.g., calculate the area of a triangle using the formula) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Calculate the volume of rectangular prisms given its dimensions (i.e., given length, width, and height) ◆ Calculate the area of the base of a solid object given dimensions (e.g., calculate the area of the base of a cylinder with a radius of 3 feet and a height of 1.5 feet) 	<ul style="list-style-type: none"> ◆ Calculate the volume of a sphere in a real-life or mathematical problem (e.g., using the formula) ◆ Calculate the volume of a cone in a real-life or mathematical problem (e.g., using the formula) ◆ Calculate the volume of a cylinder in a real-life or mathematical problem (e.g., using the formula) ◆ Calculate the volume of a pyramid in a real-life or mathematical problem (e.g., using the formula) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Visualize relationships between two-dimensional and three-dimensional objects.	This standard is not common to both high school pathways and will not be assessed.		

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Modeling with Geometry

High School		
Cluster	Standards as written	
Apply geometric concepts in modeling situations.	H.G-MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *
	H.G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *
	H.G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *
	H.G-MG.4	Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. *

* indicates Modeling standard

ENTRY POINTS for Geometry

Modeling with Geometry Standards in High School

Less Complex

More Complex

The student will:

The student will:

The student will:

Apply
geometric
concepts in
modeling
situations.

These standards are not common to both high school pathways and will not be assessed.

High School Conceptual Category – Statistics and Probability

	Standards	Entry Points	Access Skills
Interpreting Categorical and Quantitative Data	Pages 233 – 234	Pages 235 – 236	
Making Inferences and Justifying Conclusions	Page 237	Page 238	
Conditional Probability and the Rules of Probability	Pages 239 – 240	Page 241	
Using Probability to Make Decisions	Page 242	Page 243	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Interpreting Categorical
and Quantitative Data

High School

Cluster	Standards as written	
Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.	H.S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots). *
	H.S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *
	H.S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *
	H.S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. *
Summarize, represent, and interpret data on two categorical and quantitative variables.	H.S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. *
	H.S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. *
	H.S-ID.6a	Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use functions fitted to data or choose a function suggested by the context. Emphasize linear and exponential models. *
	H.S-ID.6b	Informally assess the fit of a function by plotting and analyzing residuals. *
	H.S-ID.6c	Fit a linear function for a scatter plot that suggests a linear association. *

* indicates Modeling standard

Interpret linear models.	H.S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
	H.S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
	H.S-ID.9	Distinguish between correlation and causation. ★

★ indicates Modeling standard

ENTRY POINTS for Statistics and Probability

Interpreting Categorical and Quantitative Data Standards in High School

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.	<ul style="list-style-type: none"> ◆ Represent a simple set of data on a given graphical display (e.g., dot/box plot) ◆ Order a set of numerical data from least to greatest (e.g., as a means to determine its median) ◆ Identify the minimum and maximum values in a set of data (e.g., in a data display) ◆ Calculate the range for a set of data (e.g., from a table) ◆ Identify the quartiles in a box plot (e.g., identify any/all of the values of the 5-number summary for the data) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Identify the most appropriate representation for a set of data based on a real-life situation (e.g., circle graph, bar graph, line plot, histogram, box plot, line graph, etc.) ◆ Interpret a set of data presented on a data display (e.g., find the median in a line plot) ◆ Calculate the mean, median, and mode for a set of data (e.g., from data in a table) ◆ Create a box plot given the 5-number summary for the data (e.g., class test scores, number of text messages per day, etc.) 	<ul style="list-style-type: none"> ◆ Represent the same set of data in two or more graphical representations (e.g., line plot/histogram) ◆ Create and interpret a set of data in a data display (e.g., create a line plot from a list of data and interpret the median/extreme values) ◆ Calculate the mean, median, mode, range, minimum, and maximum for a set of data (e.g., in a table) ◆ Determine which measure of center best represents a set of data (e.g., the median may be a better measure of center for a set of data that includes an outlier) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Summarize, represent, and interpret data on two categorical and quantitative variables.	<ul style="list-style-type: none"> ◆ Draw a line of best fit (trend line) (e.g., for data graphed on a scatter plot) ◆ Determine whether points on a scatter plot have a linear association (e.g., given a variety of scatterplots) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Create a two-way frequency table based on responses to a survey that asks two categorical questions (e.g., "do you like yogurt?", "do you like classical music?") ◆ Represent a set of two-variable data on a scatter plot (e.g., from data in a table or a list of ordered pairs) ◆ Use the line of best fit on a scatter plot of real-life data to predict (interpolate) likely outcomes within the range of the data (e.g., estimate a child's height at a certain age) 	<ul style="list-style-type: none"> ◆ Answer questions based on a two-way frequency table (e.g., calculate probabilities) ◆ Identify trends in a two-way frequency table (e.g., people born in Massachusetts tend to favor the Red Sox) ◆ Calculate the equation of a line of best fit drawn on a scatter plot (e.g., by finding slope and y-intercept) ◆ Use the line of best fit on a scatter plot of real-life data to predict (extrapolate) likely outcomes (e.g., a tree's height over time) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

ENTRY POINTS for Statistics and Probability

Interpreting Categorical and Quantitative Data Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Interpret linear models.	<ul style="list-style-type: none"> ◆ Determine any correlation (e.g., positive, negative, none) of data displayed on a scatter plot ◆ Determine the line of best fit for a set of data using a graphing calculator (e.g., by entering data) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Describe the strength and nature of any correlation of data in a scatter plot (e.g., strong positive, weak negative, etc.) ◆ Interpret the correlation coefficient for a set of data (e.g., $r = -0.52$ indicates a weak negative correlation) ◆ Interpret the y-intercept of the line of best fit for data in a scatter plot in terms of a context (e.g., in a graph that displays temperature between noon and 10 p.m. the y-intercept represents the temperature at noon) 	<ul style="list-style-type: none"> ◆ Interpret the relationship between two variables in a scatter plot when there is a linear relationship (e.g., math scores vs. number of days students were absent) ◆ Interpret the line of best fit for data in a scatter plot in terms of a context (e.g., ice cream sales increase as temperature rises) ◆ Interpret the slope of the line of best fit for data in a scatter plot in terms of a context (e.g., in a graph of test scores vs. hours for every hour of television watched, test scores decreased by 7 points) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Making Inferences and Justifying
 Conclusions

High School

Cluster	Standards as written	
Understand and evaluate random processes underlying statistical experiments. Use calculators, spreadsheets, and other technology as appropriate.	H.S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
	H.S-IC.2	Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of five tails in a row cause you to question the model? ★</i>
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	H.S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
	H.S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
	H.S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★
	H.S-IC.6	Evaluate reports based on data. ★

ENTRY POINTS for Statistics and Probability
Making Inferences and Justifying Conclusions Standards in High School

Less Complex

More Complex

The student will:

The student will:

The student will:

Understand and evaluate random processes underlying statistical experiments. Use calculators, spreadsheets, and other technology as appropriate.

These standards are not common to both high school pathways and will not be assessed.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

These standards are not common to both high school pathways and will not be assessed.

CONTENT AREA Mathematics

CONCEPTUAL CATEGORY Statistics and Probability

DOMAIN Conditional Probability and the Rules
of Probability

High School

Cluster	Standards as written	
Understand independence and conditional probability and use them to interpret data from simulations or experiments.	H.S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). *
	H.S-CP.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. *
	H.S-CP.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. *
	H.S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. *</i>
	H.S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. *</i>

* indicates Modeling standard

+ indicates standard is beyond College and Career Ready

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MATHEMATICS

Use the rules of probability to compute probabilities of compound events in a uniform probability model.	H.S-CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ★
	H.S-CP.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. ★
	H.S-CP.8	(+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model. ★
	H.S-CP.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems. ★

★ indicates Modeling standard

+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Statistics and Probability

Conditional Probability and the Rules of Probability Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Understand independence and conditional probability and use them to interpret data from simulations or experiments.	<ul style="list-style-type: none"> ◆ Calculate the probability of a specific outcome (e.g., rolling a die and getting “3”) ◆ List the possible outcomes (sample space) of an event (e.g., list the 36 possible outcomes of rolling 2 dice) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Distinguish between independent and dependent events in a real-life situation (e.g., choosing marbles from a bag with replacement vs. without replacement) ◆ Answer questions based on two-way frequency tables of categorical data (e.g., based on two questions asked of the same sample) 	<ul style="list-style-type: none"> ◆ Calculate conditional probability (e.g., calculate the probability of a girl liking asparagus from a two-way table of classroom data) <p><i>Continue to address skills and concepts that approach grade-level expectations in this cluster</i></p>
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	<ul style="list-style-type: none"> ◆ Create Venn diagrams given categorical data (e.g., 25 students on the field hockey team, 20 students on the softball team and 6 students on both teams) ◆ Create two-way frequency tables given categorical data (e.g., favorite pizza topping vs. favorite type of movie) <p><i>See entry points for earlier grades in this domain</i></p>	<ul style="list-style-type: none"> ◆ Calculate probabilities based on a Venn diagrams (e.g., what is the probability of an athlete playing on both the football and baseball teams) ◆ Calculate probabilities from two-way frequency tables (e.g., calculate the probability of a left-handed tuba player from data) ◆ Calculate probabilities of compound events (e.g., calculate the probability of a drawing a card that is a heart or a face card) 	<ul style="list-style-type: none"> ◆ Calculate probabilities of compound events from Venn diagrams using the Addition Rule (e.g., the probability of A, the probability of B, the probability of A and B, the probability of A or B, the probability of neither A nor B) ◆ Calculate probabilities of compound events from two-way frequency tables using the Addition Rule (e.g., the probability of students who prefer horror or adventure movies who also prefer pepperoni pizza) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Using Probability to Make Decisions

High School

Cluster	Standards as written	
Calculate expected values and use them to solve problems.	H.S-MD.1	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ★
	H.S-MD.2	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★
	H.S-MD.3	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. <i>For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. ★</i>
	H.S-MD.4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. <i>For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? ★</i>
Use probability to evaluate outcomes of decisions.	H.S-MD.5	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★
	H.S-MD.5a	(+) Find the expected payoff for a game of chance. <i>For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. ★</i>
	H.S-MD.5b	(+) Evaluate and compare strategies on the basis of expected values. <i>For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident. ★</i>
	H.S-MD.6	(+) Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). ★
	H.S-MD.7	(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, or pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater) ★

ENTRY POINTS for Statistics and Probability
Using Probability to Make Decisions Standards in High School

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Calculate expected values and use them to solve problems.	These standards are considered beyond “College and Career Ready” and will not be assessed.		
Use probability to evaluate outcomes of decisions.	These standards are considered beyond “College and Career Ready” and will not be assessed.		