Ratios and Proportional Relationships

	Standards	Entry Points	Access Skills
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CONTENT AREA Mathematics

DOMAIN Ratios and Proportional Relationships

Grade 6

Cluster	Standards as written	
Understand ratio concepts and use ratio reasoning to solve problems.	**6.RP.A.1**	Understand the concept of a ratio including the distinctions between part: part and part: whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities. *For example: The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every two wings there was one beak; For every vote candidate A received, candidate C received nearly three votes, meaning that candidate C received three out of every four votes or ¾ of all votes.*
	6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ and use rate language in the context of a ratio relationship, *including the use of units*. *For example: This recipe has a ratio of three cups of flour to four cups of sugar, so there is ¾ cup of flour for each cup of sugar; We paid $75 for 15 hamburgers, which is a rate of five dollars per hamburger.*[1]
	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
	6.RP.A.3a	Make tables of equivalent ratios relating quantities with whole-number measurements. Find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
	6.RP.A.3b	Solve unit rate problems, including those involving unit pricing, and constant speed. *For example, if it took seven hours to mow four lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
	6.RP.A.3c	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $^{30}/_{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.
	6.RP.A.3d	Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities. *For example, Malik is making a recipe, but he cannot find his measuring cups! He has, however, found a tablespoon. His cookbook says that 1 cup = 16 tablespoons. Explain how he could use the tablespoon to measure out the following ingredients: two cups of flour, ½ cup sunflower seed, and 1¼ cup of oatmeal.*[2]
	6.RP.A.3e	Solve problems that relate the mass of an object to its volume.

[1] Expectations for unit rates in this grade are limited to non-complex fractions.
[2] Example is from the Illustrative Mathematics Project:

ENTRY POINTS and ACCESS SKILLS for Ratios and Proportional Relationships Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	**ENTRY POINTS**		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Understand ratio concepts and use ratio reasoning to solve problems.	♦ Respond to materials that demonstrate ratios and proportional relationships ♦ Track materials that demonstrate ratios and proportional relationships ♦ Shift focus from materials that demonstrate ratios and proportional relationships ♦ Grasp materials that demonstrate ratios and proportional relationships ♦ Use two hands to hold materials that demonstrate ratios and proportional relationships ♦ Release materials used to demonstrate ratios and proportional relationships ♦ Orient materials used to demonstrate ratios and proportional relationships ♦ Locate objects partially hidden or out of sight (e.g., remove barrier) to expose a ratio ♦ Use one object to act on another used to demonstrate ratios	♦ Create ratios among objects (e.g., ratio of circles to squares is 5:3) ♦ Identify a part-to-part relationship in a real-life situation using a proportion (e.g., 8 boys to 3 girls or 8:3 boys to girls) ♦ Express percentages using drawings or technology (e.g., 50% = ½ of a circle) ♦ Express percentages as fraction equivalents (e.g., 75% = 75/(100)) ♦ Convert simple measurement units (e.g., feet to yards or gallons to pints) ♦ Determine whether points graphed on a coordinate plane represent a proportional relationship (e.g., points which create a line that does not pass through the origin do not represent such a relationship) ♦ Identify equivalent fractions (e.g., given 1/4, identify 2/8 and 5/20 as equivalents)	♦ Express part-to-part ratios in mathematical or real-life situations (e.g., 6 blue marbles to 8 green marbles represents a ratio of 6:8) ♦ Express the mathematical relationship of two related quantities as a ratio (e.g. in a bird the ratio of beaks to wings is 1:2 and the ratio of claws to beaks is 8:1) ♦ Plot equivalent ratios as ordered pairs, in the first quadrant of a coordinate plane (e.g. (2, 4), (3, 6), etc.) ♦ Convert measurement units from different measurement systems (e.g. kilometers to miles) ♦ Identify the factor used to obtain equivalent fractions (e.g., 1/2=3/6 because 3/3=1 and 1/2 · 3/3=3/6) ♦ Create equivalent fractions (e.g., given a visual model of 3/4 create an equivalent model of 6/8)	♦ Express part-to-whole ratios in mathematical or real-life situations (e.g., given 6 blue marbles and 8 green marbles, 6/14 of the marbles are blue) ♦ Calculate a unit rate between two given quantities (e.g., if 16 cupcakes are split between 8 children then unit rate is 2 cupcakes per child; 5 apples cost \$2.00 so the unit rate is \$0.40 per apple) ♦ Calculate unit rates in real-life problems to make comparisons (e.g., compare prices from Home Depot and Lowes by calculating unit rates) ♦ Convert rates by manipulating measurement units (e.g., miles per hour to feet per second) ♦ Match a fraction to its equivalents (e.g., given 1/3 and a bank of fractions identify equivalents such as 2/6 and/or 4/12)

ENTRY POINTS and ACCESS SKILLS for Ratios and Proportional Relationships Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	The student will:	ENTRY POINTS The student will:	The student will:
Understand ratio concepts and use ratio reasoning to solve problems.	♦ Turn on/off technology used to demonstrate ratios and proportional relationships (e.g., turn on voice-generating device to describe a relationship using “to/for every” language) ♦ Imitate action to create proportional relationships ♦ Initiate cause-and-effect response (e.g., turn on technology tool to activate ratio computer program) ♦ Sustain ratio and proportional relationship activity through response ♦ Gain attention during a ratio activity ♦ Make a request during ratio activity ♦ Choose materials to be distributed in a ratio and proportional relationship activity ♦ Attend visually, aurally, or tactilely to materials that demonstrate ratios and proportional relationships		♦ Calculate unit rates in real-life problems from a table showing a proportional relationship (e.g., determine the price of one item from a table showing various numbers and total costs of the item)	♦ Create equivalent fractions using numbers (e.g., given a fraction and another with a missing numerator or denominator name the unknown number)

CONTENT AREA Mathematics

DOMAIN Ratios and Proportional Relationships

Grade 7		
Cluster	**Standards as written**	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	**7.RP.A.1**	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*
	7.RP.A.2	Recognize and represent proportional relationships between quantities.
	7.RP.A.2a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
	7.RP.A.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
	7.RP.A.2c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
	7.RP.A.2d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
	7.RP.A.3	Use proportional relationships to solve multi-step ratio, rate, and percent problems. *For example, simple interest, tax, price increases and discounts, gratuities and commissions, fees, percent increase and decrease, percent error.*

ENTRY POINTS for
Ratios and Proportional Relationships Standards in Grade 7

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Analyze proportional relationships and use them to solve real-world and mathematical problems.	♦ Calculate a unit rate in the context of a described proportional relationship (e.g., a recipe that calls for 4 cups of flour and 2 cups of sugar has a ratio of 2 cups of flour to 1 cup of sugar) ♦ Find equivalent ratios for a given ratio in a real-life situation (e.g., if the ratio of girls to boys is 2:3, then for 40 girls there are 60 boys, and for 30 girls there are 45 boys) ♦ Calculate the percentage in a percent problem (e.g., what percent of 60 is 45?) ♦ Calculate the base in a percent problem (e.g., 9 is 75% of what number?) ♦ Calculate the unknown amount in a percent problem (e.g., what number is 50% of 8?) ♦ Solve one-step equations using multiplication (e.g., 3x=45 or 4x=36) ♦ Determine the amount of tax charged on an item given the item cost in whole dollars and the tax rate (e.g., the cost of an item is $10, and the tax rate is 4%, the amount of tax charged is $0.40) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Complete a table of a proportional relationship by filling in missing values (e.g., in a table, x values of 0, 3, _, 9, 18, and y values of 0, 2, 4, _, 12) ♦ Compute unit rates in terms of distance and time (e.g., If Jaycee traveled 20 miles in 30 minutes, then she traveled at a rate of 40 miles per hour) ♦ Solve percentage proportions where one missing quantity is represented by a variable (e.g., 8/40=x/100) ♦ Create a proportion table given a ratio (e.g., given the ratio 1:3, create a table of equivalent ratios such as 2:6, 3:9, 6:18, 12:36, etc.) ♦ Identify the unit rate from a table, graph, equation, or a description (e.g., if 4 pounds of grapes cost $9, determine is the cost of one pound) ♦ Determine whether values in a table represent a proportional relationship (e.g., do the values (3, 5), (6, 16), (15, 25) and (25, 75) in table form represent a proportion?) ♦ Determine the amount of tax charged on an item given the item cost and the tax rate (e.g., the cost of an item is $18.99, and the tax rate is 5%, the amount of tax charged is $0.95) ♦ Convert fractions to their percentage equivalent (e.g., convert 80/100 to 80%, ½ to 50%, and 3/5 to 60%) ♦ Convert fractions to their decimal equivalent (e.g., convert 70/100 to 0.7, 1/8 to 0.125, and 15/40 to 0.375)	♦ Solve proportions where one missing quantity is represented by a variable (e.g., 3/5=x/15) ♦ Solve percentage decrease or increase problems (e.g., a dress that originally cost $50 was on sale for 60% off; a train fare of $14 was increased by 20%) ♦ Compute unit rates in a variety of contexts (e.g., miles per hour, cents per stick of gum, minutes per commercial) ♦ Solve equations that represent proportional relationships in real life (e.g., use $d = rt$ to find time if distance is 120 miles and the rate is 60 miles per hour) ♦ Solve mixed single-step percent problems using proportional relationships (e.g., what is 45% of 80?; 35 percent of what is 7?; 60 is what percent of 200?) ♦ Determine the total cost of an item given the item cost and the tax rate (e.g., the cost of an item is $11.95 and the tax rate is 6%, the total cost of the item is $12.67) ♦ Determine the tax rate given the pre-tax and the total costs (e.g., the cost of an item is $599.00 and the total cost with tax is $628.95, the tax rate is 5%) ♦ Solve word problems involving percentages that increase and/or decrease (e.g., sales price, mark up, sales tax)

Number and Operations – Fractions

	Standards	Entry Points	Access Skills
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CONTENT AREA Mathematics

DOMAIN Number and Operations–Fractions

Grade 3		
Cluster	**Standards as written**	
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.	**3.NF.A.1**	Understand a fraction $^1/_b$ as the quantity formed by 1 part when a whole (a single unit) is partitioned into *b* equal parts; understand a fraction $^a/_b$ as the quantity formed by *a* parts of size $^1/_b$.
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.
	3.NF.A.2a	Represent a unit fraction, $^1/_b$, on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $^1/_b$ and that the fraction $^1/_b$ is located $^1/_b$ of a whole unit from 0 on the number line.
	3.NF.A.2b	Represent a fraction $^a/_b$ on a number line diagram by marking off *a* lengths $^1/_b$ from 0. Recognize that the resulting interval has size $^a/_b$ and that its endpoint locates the number $^a/_b$ on the number line.
	3.NF.A.3	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.
	3.NF.A.3a	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
	3.NF.A.3b	Recognize and generate simple equivalent fractions, e.g., $^1/_2 = ^2/_4$, $^4/_6 = ^2/_3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
	3.NF.A.3c	Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
	3.NF.A.3d	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

ENTRY POINTS and ACCESS SKILLS for Number and Operations–Fractions Standards in Grade 3

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	ENTRY POINTS The student will:	The student will:	The student will:
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.	♦ Respond to materials that demonstrate objects that can be divided into equal parts ♦ Track materials that demonstrate that objects can be divided into equal parts ♦ Shift focus from materials that demonstrate that objects can be divided into equal parts ♦ Grasp materials that demonstrate that objects can be divided into equal parts ♦ Use two hands to hold materials that demonstrate that objects can be divided into equal parts ♦ Release materials that demonstrate that objects can be divided into equal parts ♦ Move materials that demonstrate that objects can be divided into equal parts	♦ Explain what the numerator and denominator of a fraction represent (e.g., in 2/4, the 4 tells you how many parts the whole is divided into and 2 is the number of parts you have and use a drawing to illustrate) ♦ Identify concepts of whole and 1/2 using manipulatives and/or familiar objects (e.g., using sets of objects or shapes with shaded parts, identify 1/2 and whole) ♦ Partition a whole into 1/2, 1/3, or 1/4 equal parts using visual models, number lines, or manipulatives (e.g., given a rectangle, draw lines to divide it into 3 equal parts representing 1/3) ♦ Compare fractions of the same whole to determine which is greater (e.g., show a number line with points at ¼ and ¾ and ask which is greater)	♦ Compare visual representations of fractions using the terms "greater than," "less than," or "equal to" (e.g., verbalize that ½ is greater than ¼) ♦ Match visual representations of simple fractions to the name of the fraction (e.g., given visual fraction models with sections already shaded in, identify amounts such as ¾, ½, $^{5}/_{8}$) ♦ Compare parts of a whole (quarters, thirds, halves) to determine relative size of each (1/2, 1/3, 1/4) using manipulatives or visual models (e.g., use manipulatives to show that ½ > 1/3) ♦ Label unit fractions* on a number line (e.g., given a number line labeled with 0 and 1 and divided into 6 equal parts, plot and label a point at 1/6)	♦ Identify parts of a whole using visual fraction models (e.g., using shapes divided into equal parts and having one part shaded in, identify 1/2, 1/3, 1/4, 1/6, 1/8) ♦ Divide a number line into equal parts and label points (e.g., mark off a number line labeled with 0 and 1 into 6 equal sections and label each point with 1/6, 2/6, 3/6, etc.) ♦ Record results of the comparisons of two fractions with like denominators or like numerators using symbols (e.g., use the symbols <, =, or > to write a comparison of 1/4 and ¾, or 2/4 and 2/8) ♦ Create a visual representation of simple fractions (e.g., divide a shape into 4 equal parts and shade in ¾ recognizing that the parts do not need to be touching)

ENTRY POINTS and ACCESS SKILLS for
Number and Operations–Fractions Standards in Grade 3

Less Complex ⟵⟶ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS** The student will:	The student will:	The student will:
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8. (continued)	♦ Orient materials that demonstrate that objects can be divided into equal parts ♦ Locate objects partially hidden or out of sight (e.g., remove barrier to expose part that when added to object equals the whole object) ♦ Turn device on/off to participate in an activity on fractions (e.g., turn on voice-generating device) to comment on fraction activity ♦ Imitate action required to divide object ♦ Initiate cause-and-effect response (e.g., turn on technology tool) to activate fraction activity ♦ Sustain activity through response in a fraction-based activity ♦ Gain attention (e.g., request a turn) with fraction materials ♦ Make a request in a fraction based activity	♦ Answer questions about fractions (e.g., Show a shaded figure that represents 3/6 and answer "Does this show the fraction 3/6 or ¾?") ♦ Match a fraction represented visually to the fraction represented numerically or verbally (e.g., a fraction model) ♦ Create equivalent fractions using manipulatives (e.g., using two equivalent wholes made up of manipulatives, show that 2/4 = ½ with the appropriate shapes) ✱ **Unit Fraction**: a fraction with a numerator of one	♦ Partition a number line labeled with 0 and 1 into 2, 4, or 8 equal parts (e.g., draw hash marks on the number line to divide the length from 0 to 1 into 4 equal parts) ♦ Order simple fractions by plotting and labeling points on two number lines that have already been divided into equal parts (e.g., plot points at 5/8 on one line and 1/2 on another line to show that 5/8>1/2) ♦ Express whole numbers as fractions using models and show they are equal to 1(e.g., divide two equivalent shapes into 4 equal parts and 6 equal parts to show that 4/4 = 6/6 = 1 as long as the wholes are equal in size) ♦ Match a visual representation of a fraction to a fractional number line (e.g., match a shape with 1/4 shaded to a number line with a point at 1/4)	♦ Order simple fractions on a number line that has already been divided into equal parts (e.g., plot points at 2/8 and 1/2 on a number line divided into 8 sections to show that 2/8<1/2) ♦ Label points with simple fractions on a number line (e.g., label given points at 1/6, 3/6, 5/6 on a number line) ♦ Determine the number of unit fractions* in a whole by using same sized pieces to create a whole (e.g., If you need 3 pieces to make a whole then each piece represents the unit fraction* 1/3) ♦ Express whole numbers as fractions and fractions as whole numbers using models and show they are equivalent (e.g., given two equivalent shapes divided into 4 equal parts, show that 4/4 = 1 and 8/4 = 2) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS** The student will:
Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8. (continued)	♦ Choose from an array of two in a fraction-based activity (e.g., choose materials to be divided into equal parts) ♦ Attend visually, aurally, or tactilely to materials that demonstrate objects that can be divided into equal parts	

CONTENT AREA Mathematics

DOMAIN Number and Operations–Fractions

Grade 4

Cluster	Standards as written	
Extend understanding of fraction equivalence and ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	**4.NF.A.1**	Explain why a fraction $^{a}/_{b}$ is equivalent to a fraction $^{(n \times a)}/_{(n \times b)}$ by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.
	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $^{1}/_{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	**4.NF.B.3**	Understand a fraction $^{a}/_{b}$ with $a > 1$ as a sum of fractions $^{1}/_{b}$..
	4.NF.B.3a	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (The whole can be a set of objects.).
	4.NF.B.3b	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models. Examples: $^{3}/_{8} = {}^{1}/_{8} + {}^{1}/_{8} + {}^{1}/_{8}$; $^{3}/_{8} = {}^{1}/_{8} + {}^{2}/_{8}$; $2\ {}^{1}/_{8} = 1 + 1 + {}^{1}/_{8} = {}^{8}/_{8} + {}^{8}/_{8} + {}^{1}/_{8}$.
	4.NF.B.3c	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
	4.NF.B.3d	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using drawings or visual fraction models and equations to represent the problem.
	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
	4.NF.B.4a	Understand a fraction $^{a}/_{b}$ as a multiple of $^{1}/_{b}$.. *For example, use a visual fraction model to represent* $^{5}/_{4}$ *as the product* $5 \times (^{1}/_{4})$*, recording the conclusion by the equation* $^{5}/_{4} = 5 \times (^{1}/_{4})$*.*
	4.NF.B.4b	Understand a multiple of $^{a}/_{b}$ as a multiple of $^{1}/_{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

	4.NF.B.4c	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
Understand decimal notation for fractions, and compare decimal fractions.	**4.NF.C.5**	Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
	4.NF.C.6	Use decimal notation to represent fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

ENTRY POINTS for Number and Operations–Fractions Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Extend understanding of fraction equivalence and ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	♦ Distinguish between equal and non-equal parts of a whole (e.g., compare unit fractions* or fractions with like denominators but unlike numerators using models or number lines) ♦ Identify a fraction 1/*b* as the quantity formed by one part when a whole is partitioned into *b* equal parts (e.g., given several shapes with unit fractions* shaded in, identify the parts of the whole as ½, ¼, 1/8, etc.) ♦ Demonstrate fractions equivalent to ½ using fraction models, manipulatives and/or technology (e.g., show that 2/4 = ½ on a number line) ♦ Compare two fractions with like denominators by comparing their relative size (e.g., fraction models) ♦ Identify which of two fractions represents a larger part of a whole using fraction models or manipulatives (e.g., given two number lines marked from 0 to 1 with points plotted for 4/5 and 3/8, determine that 4/5 is larger because it is more of the whole because it is to the right of 3/8) ♦ Compare two fractions with unlike denominators by comparing their relative size using fraction models	♦ Identify equivalent fractions using fraction models, manipulatives, and/or technology (e.g., given several rectangles already shaded in with different fraction amounts, show that 2/6 = 1/3) ♦ Compare two fractions with like denominators, represented numerically, using >, <, or = (e.g., 5/6 > 3/6) ♦ Determine which of two fractions with like denominators represents a larger part of a whole by representing the fractions with fraction models or manipulatives (e.g., given two congruent rectangles divided into eighths, shading in one to represent 3/8 and another to represent 7/8, and then identifying which is more of the whole)	♦ Generate multiple pairs of equivalent fractions using fraction models, manipulatives and/or technology (e.g. show that 2/4 = 3/6 by drawing two congruent rectangles with one divided into fourths and one divided into sixths and shading in 2/4, 3/6, 4/8 to show they are equivalent areas) ♦ Compare two fractions with unlike denominators by demonstrating which is greater or less than the benchmark of ½ using fraction models, manipulatives or technology (e.g., showing on two number lines that 2/8 < 3/4 because 2/8 is to the left of ½ on the number line and 3/4 is to the right of ½) ♦ Demonstrate, using fraction models or manipulatives, that the whole is equal to the sum of the partitioned parts (e.g., 4/4 = 1; 1 = 8/8) ♦ Compare visual models of fractions with unlike denominators using symbols (<, >, or =) (e.g., given two models, one of 4/5 and one with 3/10, determine which is greater and then write 4/5 > 3/10) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	♦ Add unit fractions* with like denominators with only two addends using fraction models (e.g., using a number line or shapes, show that ¼ + ¼ = 2/4 or ½ + ½ = 2/2 or 1) ♦ Add simple fractions using visual models, manipulatives, or technology (e.g., showing that two halves equal a whole or two fourths equal a half) ♦ Subtract simple fractions using visual models, manipulatives or technology (e.g. using manipulatives, show 3/4 – 1/4 = 2/4 or 1/2) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add unit fractions* with like denominators with more than two addends using both fraction models and equations (e.g., using a number line or shapes, show that ¼ + ¼ + ¼ = ¾ or ½ + ½ + ½ =3/2 and then write the equation) ♦ Add and subtract fractions with like denominators using visual fraction models (e.g. use a rectangle divided into 12 equal parts to solve 2/12 + 3/12 by shading 2 parts and then 3 parts to find the total number of twelfths) ♦ Multiply a fraction by a whole number using visual models and repeated addition (e.g., showing that ¼ x 3=¾ and ¼ + ¼ + ¼ = ¾) ✱ **Unit Fraction**: a fraction with a numerator of one	♦ Decompose a fraction into a sum of unit fractions* with the same denominator one way (e.g., 3/8 = 1/8 + 1/8 + 1/8) ♦ Subtract fractions with like denominators using both fraction models and equations (e.g., show 5/8 – 3/8 = 2/8 using a number line marked with eighths to "hop" three eighths to the left from 5/8 to end up at 2/8 and then write the equation) ♦ Add and subtract fractions and mixed numbers with like denominators when mixed numbers do not need to be re-written as fractions (e.g. 4 ¾ + ¼ = 5, 2 4/5 – 3/5 = 2 1/5) ♦ Solve word problems involving addition and subtraction of no more than two fractions with like denominators using fraction models, manipulatives, or technology (e.g., Jamie has 1/8 cup of apple juice and 4/8 cup of pineapple juice. What is the total amount of juice, in cups, that Jamie has in all?) ♦ Represent a mixed number as an equivalent fraction using fraction models, manipulatives, or technology, given the written form (e.g., show that 1½ = 3/2)

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (continued)			♦ Multiply a non-unit fraction by a whole number using visual models, manipulatives, or technology (e.g., 3 X 2/4 =2/4 + 2/4 + 2/4 also equals 3 groups of 2/4 and each 2/4 = ¼ + ¼) ♦ Solve a multiplication word problem involving multiplying a fraction by a whole number using visual models, manipulatives, or technology (e.g., Danny rode his bike 3/8 mile each day in the summer. What is the total distance Danny rode over 5 days? 5 X 3/8 = 15/8 or 1 7/8 miles) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Understand decimal notation for fractions, and compare decimal fractions.	♦ Order decimals on a number line (e.g., given a number line labeled with tenths in decimals, plot points at 0.4, 0.7, and 0.8 or given a number line labeled with hundredths in decimals, plot points at 0.62, 0.64, and 0.68) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Show that a fraction with a denominator of ten is equivalent to a fraction with a denominator of 100 by using visual models, manipulatives, or technology (e.g., using base-ten blocks, show that 5/10 = 50/100) ♦ Compare two decimals up to the hundredths by reasoning about their size using symbols (<, >, =) or visual model	♦ Express a fraction with a denominator of ten as an equivalent fraction with a denominator of 100 (e.g., 3/10 = 30/100 or 60/100 = 6/10) ♦ Use decimal notation for fractions with denominators of ten. (e.g., 2/10=0.2) ♦ Compare two decimals to the tenths by reasoning about their size using symbols (<, >, or =) or visual model (e.g., use a number line to show that 0.65 > 0.40 because 0.40 is to the left of 0.65 and write the inequality) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics

DOMAIN Number and Operations–Fractions

Grade 5		
Cluster	**Standards as written**	
Use equivalent fractions as a strategy to add and subtract fractions.	**5.NF.A.1**	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.).*
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	**5.NF.B.3**	Interpret a fraction as division of the numerator by the denominator ($^{a}/_{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when three wholes are shared equally among four people each person has a share of size ¾. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
	5.NF.B.4a	Interpret the product ($^{a}/_{b}$) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model and/or area model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15 . (In general, (a/b) × (c/d) = ac/bd.)*
Apply and extend previous understand-ings of multiplication and division to	**5.NF.B.4b**	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
	5.NF.B.5a	Interpret multiplication as scaling (resizing), by: comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, without multiplying tell which number is greater: 225 or ¾ x 225; 11/50 or 3/2 x 11/50?*

multiply and divide fractions. (continued)	**5.NF.B.5b**	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $^a/_b = (n \times a)/(n \times b)$ to the effect of multiplying $^a/_b$ by 1.
	5.NF.B.6	Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. [1]
	5.NF.B.7a	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
	5.NF.B.7b	Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*
	5.NF.B.7c	Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if three people share ½ lb. of chocolate equally? How many $^1/_3$-cup servings are in two cups of raisins?*

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

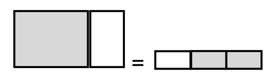
ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 5

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use equivalent fractions as a strategy to add and subtract fractions.	♦ Add fractions with like denominators creating sums greater than/or equal to one (e.g., 7/10 + 4/10 = 11/10) ♦ Subtract fractions with like denominators creating differences less than one (e.g., 6/8 – 3/8 = 3/8) ♦ Identify two equivalent fractions with unlike denominators that are represented by fraction models (e.g. given two fraction models referring to the same whole, show that they are ¾ and 6/8 and are equivalent) ♦ Identify visual fraction models that represent mixed numbers (e.g., identify a fraction model consisting of 2 rectangles with 1 and ¾ shaded in as representing 1¾) ♦ Compare two fractions with like numerators or like denominators by reasoning about their size (e.g., explain that 3/8 < 3/6 because eighths are a smaller size than sixths so if you have 3 sixths of a pizza you have more than if you have 3 eighths of the same size pizza) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add unit fractions* with unlike denominators, by using manipulatives or technology to create equivalent fractions with like denominators (e.g., 1/6 + ¼ = 2/12 + 3/12 = 5/12) ♦ Subtract unit fractions* with unlike denominators, by using manipulatives or technology to create equivalent fractions with like denominators (e.g., 1/4 – 1/8 = 2/8 – 1/8 = 1/8) ♦ Represent mixed numbers with fraction models (e.g., draw a fraction model to represent 3/2 and show that 3/2 = 1½) ♦ Represent word problems with fractions with like denominators (e.g., represent a word problem that requires the student to add 1/8 + 5/8 with fraction models referring to the same whole) ♦ Compare two fractions with different numerators and different denominators (e.g., ½ < 5/8 using technology, or ½ is equal to 6/12 using a fraction model) ♦ Use benchmark fractions to compare fractions with like denominators using visual fraction models, manipulatives, or technology (e.g. know that 7/10 > ½ and 4/9 < ½ so therefore 7/10 > 4/9)	♦ Add fractions with unlike denominators, creating sums less than one by using manipulatives or technology to create equivalent fractions with like denominators (e.g., 2/5 +1/3 =6/15 + 5/15 = 11/15) ♦ Subtract fractions with unlike denominators, creating differences less than one by using manipulatives or technology to create equivalent fractions with like denominators (e.g., 4/5 – 2/3 = 12/15 – 10/15 = 2/15) ♦ Add and subtract mixed numbers with like denominators using manipulatives or technology (e.g., 1 1/3 + 2 2/3 = 3 3/3 = 4 or 2 ¼ - 1 ¾ = 9/4 – 7/4 = 2/4) ♦ Solve word problems involving addition or subtraction of fractions with like denominators using manipulatives or technology (e.g., draw fraction models to represent the fractions and the solution) ♦ Estimate sums or differences of fractions with like denominators, use benchmark fractions and number sense (e.g. knowing that 3/5 + 4/5 =7/10 is false because 3/5 and 4/5 are both greater than ½ so the sum must be greater than 1 and 7/10 is less than 1)

ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 5

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use equivalent fractions as a strategy to add and subtract fractions. (continued)		♦ Show valid comparisons of fractions that refer to the same whole (e.g., given two pictures of wholes divided into 6ths and 12ths, show that 2/6 is equal to 4/12)	♦ Show that comparisons are valid only if the fractions refer to the same whole (e.g., given pictures of pairs of fractions, some with different size wholes and some with the same size wholes, be able to choose the correct comparisons:  is not true because the wholes are not the same size) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	♦ Show/express that a unit fraction* is represented by the division of the numerator by the denominator (e.g., show that 1/5 means dividing one whole into 5 equal parts) ♦ Write a multiplication problem involving a whole number and a fraction as a repeated addition problem (e.g., 4 X 3/5 = 3/5 + 3/5 + 3/5 + 3/5) ♦ Show that multiplying a fraction by a fraction is similar to creating a model of the first fraction, then scaling each part by the other fraction (e.g. ¼ X ½ can be modeled by describing or drawing a whole rectangle divided into 4 equal parts, and then each of the 4 parts(1/4) is divided into 2 equal parts so that each part is now 1/8 of the whole rectangle)	♦ Match fractions with their equivalent division expressions (e.g. 3/8 = 3 ÷ 8, not 8 ÷ 3 nor 3 X 8) ♦ Represent connections between fractions and division with the use of visual models, manipulatives or technology (e.g., show 8/4 can be represented as 8 candy bars into 4 groups results in each group getting 2 candy bars) ♦ Multiply a whole number by a unit fraction* using a number line marked from 0 to 1(e.g. use a number line labeled with ¼'s to find 3 X ¼ = ¾) ♦ Compare products of fractions and whole numbers based on the multiple using a visual fraction model (e.g., is 5 X 2/5 greater or less than 5 X 4/5 or is 3/5 X 1/2 greater or less than 7/6 X 1/2)	♦ Identify division in a real-world problem as a fraction (e.g., write "5 cookies divided equally by 10 people means each person gets 5/10 or ½ of a cookie") ♦ Multiply fractions by fractions using manipulatives, visual models and/or technology (e.g., 2/4 X 4/5 = 8/20) ♦ Multiply a whole number by a fraction less than 1 using visual models or manipulatives (e.g., show 2/3 X 4 = 8/3 by using 4 shapes that are determined to be 2/3 of a whole or by defining a whole and using 2 one-third shapes as the 2/3 and compiling 4 sets of the 2/3)

ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 5

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (continued)	♦ Solve real-world problems involving division of a whole into equal parts (e.g., divide a whole candy bar to share with four friends) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Show that dividing a fraction by a whole number will give a quotient that is less than the original using fraction models or manipulatives (e.g. ½ ÷ 3 will be equal to a fraction that is less than ½) ♦ Show that dividing a whole number by a fraction will give a quotient that is greater than the original whole number using fraction models or manipulatives (e.g., 3 ÷ ½ will be greater than 3) ✱ **Unit Fraction**: a fraction with a numerator of one	♦ Compare the size of a product to the size of one factor (with one factor being a fraction and one factor a whole number) without performing the multiplication (e.g., know that 2/3 X 4 < 4 because 2/3 < 1 so the product must be less than 1 X 4) ♦ Solve real-world problems by multiplying fractions or mixed numbers using manipulatives, visual models, and/or technology (e.g., solve "There are 4 students who need paint for their art project. Each student needs 1¾ gallons of paint. What is the total amount of paint, in gallons, needed by these four students?" by using a visual fraction model to show 1¾ four times to find solution) ♦ Solve real-world problems by dividing a whole number by a unit fraction* or a unit fraction by a whole number (e.g., 3 pizzas are each divided into fourths so there are 12 pieces, 3 ÷ 1/4 = 12) ♦ Connect division and multiplication of fractions (e.g., know that 5 ÷ ½ = 10 because 10 X ½ = 5) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Functions

	Standards	Entry Points	Access Skills
8	Page 110	Pages 111 – 113	Pages 111 – 114

CONTENT AREA Mathematics
DOMAIN Functions

Grade 8		
Cluster	**Standards as written**	
Define, evaluate, and compare functions.	**8.F.A.1**	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
	8.F.A.3	Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*
Use functions to model relationships between quantities.	**8.F.B.4**	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

[1] Function notation is not required in grade 8.

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	ENTRY POINTS		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Define, evaluate, and compare functions.	♦ Respond to materials being compared by quantity or size ♦ Track materials being compared by quantity or size ♦ Shift focus from materials being compared by quantity or size to speaker ♦ Grasp materials being compared by quantity or size ♦ Release materials being compared by quantity or size ♦ Give materials being compared by quantity or size ♦ Move materials being compared by quantity or size ♦ Orient materials being compared by quantity or size ♦ Construct two objects using materials from two sets of materials being compared by quantity or size (e.g., build two block towers, with one set of 3 blocks and one with 5 blocks) ♦ Turn on voice-generating device to participate in an activity to compare materials by quantity or size	♦ Answer yes/no questions about functions ♦ Identify the input (x) and the output (y) in an input-output table and interpret the relationship between one variable and another variable in a table (e.g., number of hours, number of miles traveled) ♦ Create an input-output table when given the input values and the function rule ♦ Graph a function on the coordinate plane using a completed table ♦ Represent unknown number quantities in an input-output table, given an input number and a rule (e.g., input = 60, rule is subtract 15, then output = 45) ♦ Answer yes/no questions about input/output tables ♦ Generate a number pattern given an addition rule and an initial value (e.g., start with 6, rule is add 4, find the next 5 numbers in the pattern)	♦ Identify linear and non-linear functions from given tables by graphing them ♦ Explain why the relationships are or are not functions in given tables (e.g., given a table with x-input: 9, 9, 16, 16 and corresponding y-output: 3, -3, 4, -4 determine the relationship is not a function because some inputs have more than one output) ♦ Explain why the relationships are or are not functions in given graphs (e.g., a graph of a circle is not a function because for some x-inputs there is more than one y-output; a horizontal line on the coordinate plane is a function because for every x input there is only one y output) ♦ Determine the addition rule in an input-output table (e.g., if input is 20 and output is 25, what is the rule?)	♦ Explain why the relationships are or are not functions in given algebraic equations (e.g., the equation y=x is a function because each input has only one output and $y^2 + x^2 = 25$ is not a function because one input has 2 outputs) ♦ Compare *slopes* of two functions presented in different ways (e.g., the algebraic representation of the function y = 3x + 2 has a positive slope and the table representation of the function X input 0, 1, 2 and corresponding y outputs 2, 0, -2 has a negative slope) ♦ Determine whether a function is linear or non-linear by graphing it ♦ Describe how varying the rate of change in a variable affects the outcome in a table (e.g., increasing the speed decreases the time needed to arrive at a destination)

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	ENTRY POINTS The student will:	The student will:	The student will:
Define, evaluate, and compare functions. (continued)	♦ Imitate action with materials being used to compare quantity or size ♦ Sustain activity comparing objects by size or quantity through response ♦ Gain attention in activity comparing objects by size and quantity (e.g., raise hand during comparison lesson on white board) ♦ Imitate action using materials compared by quantity or size ♦ Initiate cause-and-effect using materials compared by quantity or size ♦ Make a request in an activity comparing materials by size or quantity ♦ Choose from an array of two in an activity to compare materials by quantity ♦ Choose beyond an array of two to compare materials by quantity ♦ Attend visually aurally, or tactilely to materials being compared by quantity or size	♦ Generate a number pattern given a subtraction rule and an initial value (e.g., start with 50, rule is subtract 7, find the next 5 numbers in the pattern) ♦ Compare *initial values* of two functions presented in different ways (e.g.: Compare the initial value in Sam and Tom's savings account in the description and equations below: <u>description for Sam</u>: Sam deposits $20 in the bank to open an account. He puts $10 into the account each month and withdraws nothing. <u>Equation for Tom</u>: s= 4m+ 200 Answer: Sam's initial value is $20, and Tom's is $200. Tom has a greater initial value) ♦ Classify graphs of functions as linear or nonlinear given two graphical representations	♦ Determine the subtraction rule in an input-output table (e.g., if input is 60 and output is 45, what is the rule?) ♦ Generate a number pattern given a multiplication rule and an initial value (e.g., start with 2, rule is multiply by 3, find the next 4 numbers in the pattern) ♦ Compare rates of change of two functions presented in different ways (e.g.: Compare the rate of change in Sam and Tom's savings in the description and equations below. Description for Sam: Sam deposits $20 in the bank to open an account. He puts $10 into the account each month and withdraws nothing. Equation for Tom: s= 4m+ 200 Answer: Sam puts $10 in his account every month and Tom puts $4 in his account every month, so Sam's saving has a greater rate of change)	♦ Express addition and subtraction equations to represent the relationship between two variables using graphs or tables (e.g., Input -15 = Output) ♦ Determine the multiplication rule in an input-output table (e.g., if input is 3 and output is 12, what is the rule?) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	ENTRY POINTS		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Use functions to model relation-ships between quantities.	♦ Respond to materials to create graphs ♦ Track materials used to create graphs ♦ Shift focus from materials used to create graphs to speaker ♦ Grasp materials used to create graphs ♦ Use two hands to hold materials used to create graphs ♦ Release materials used to create graphs ♦ Move materials used to create graphs ♦ Orient materials used to create graphs (e.g., orient icon pictures used to label axis) ♦ Manipulate objects used to create graphs ♦ Locate objects partially hidden or out of sight needed to create a graph (e.g., remove barrier to expose materials) ♦ Use one object to act on another to create graphs (e.g., use scissors to cut materials)	♦ Determine the initial value (y-intercept) and the rate of change (slope) from graphs ♦ Match given descriptions to a given graph (e.g., match segments on a graph to segments of the following description: Jenny is walking to Samantha's house at a constant rate. Jenny gets to Samantha's house and is waiting. Jenny and Samantha ride the bus to school. The bus is moving at a constant rate but much faster than Jenny's walking rate.) ♦ Answer yes/no questions about graphs	♦ Determine the initial value (y-intercept) from graphs and linear equations ♦ Determine the rate of change (slope) from graphs and linear equations ♦ Describe a graph of a function that has labeled sections (e.g., Describe that Part A rises gently, Part B is short but flat, Part C rises like part A, but is steeper, etc.)	♦ Determine the initial value (y-intercept) from tables, graphs, linear equations, and descriptions ♦ Determine the rate of change (slope) from tables, graphs, linear equations and descriptions ♦ Sketch a graph from given context (e.g., given the context "Jenny is walking to Samantha's house at a constant rate" the student draws a straight line on the graph with a slope. And labels time on the x-axis and distance on the y-axis. When the context changes, "Jenny gets to Samantha's house and is waiting" the student draws a horizontal line, etc." *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Use functions to model relationships between quantities. (continued)	♦ Turn on technology used to create graphs ♦ Imitate action used to create graphs (e.g., imitate classmate attaching icon to graph) ♦ Initiate cause-and-effect response to turn on technology tool to activate graphing computer program ♦ Sustain graphing activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention in a graphing activity (e.g., raise hand vocalize) ♦ Make a request during a graphing activity (e.g., request a turn) ♦ Choose from materials to be used in graphing activity ♦ Attend visually, aurally, or tactilely to materials to create graphs	

High School Conceptual Category – Functions

	Standards	Entry Points	Access Skills
Interpreting Functions	Pages 201 – 202	Pages 203 – 205	
Building Functions	Pages 206 – 207	Page 208	
Linear, Quadratic, and Exponential Models	Page 209	Pages 210 – 211	
Trigonometric Functions	Page 212	Page 213	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Interpreting Functions

High School

Cluster	Standards as written	
Understand the concept of a function and use function notation.	**H.F-IF.1**	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
	H.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *For example, given a function representing a car loan, determine the balance of the loan at different points in time.*
	H.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.
Interpret functions that arise in applications in terms of the context (linear, quadratic, exponential, rational, polynomial, square root, cube root, trigonometric, logarithmic).	**H.F-IF.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ⋆
	H.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ⋆
	H.F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ⋆
Analyze functions using different representa-tions.	**H.F-IF.7**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ⋆
	H.F-IF.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima. ⋆
	H.F-IF.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ⋆

⋆ indicates Modeling standard

H.F-IF.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ★
H.F-IF.7d	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★
H.F-IF.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★
H.F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
H.F-IF.8a	Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
H.F-IF.8b	Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
H.F-IF.9	Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one polynomial function (including quadratic functions) and an algebraic expression for another, say which has the larger/smaller relative maximum and/or minimum.*
H.F-IF.MA10	Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

★ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions
Interpreting Functions Standards in High School

Less Complex ⟵⟶ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand the concept of a function and use function notation.	♦ Complete an input-output table when given the function rule and some values (e.g., student fills in missing values in a table) ♦ Identify the quantitative dependent and independent variable in a real-life situation (e.g., the number of students going on a trip (independent) and the number of buses required (dependent)) ♦ Determine whether a relationship illustrated by a set of ordered pairs or a table, involving a domain (input) and range (output) represents a function (e.g., is the relationship represented by the ordered pairs (3,4) (5,12) (3,8) a function?) ♦ Evaluate a function for a value of its domain (e.g., for the function $f(x) = -2x$ find $f(8)$) ♦ Determine the initial value (where $x = 0$) of a function in the form $f(x) = mx + b$ (e.g., identify the y-intercept of the function) ♦ Determine the rate of change of a function from its graph (e.g., by calculating slope) ♦ Determine the rate of change of a function from a table of ordered pairs (e.g., by calculating rate of change between two ordered pairs) *See entry points for earlier grades in this domain*	♦ Determine whether the relationship shown in an a list of ordered pairs, an input-output table, or a mapping represents a function (e.g., using a variety of representations) ♦ Evaluate a function for multiple values of its domain (e.g., for the function $f(x) = 2x + 1$, find $f(-1)$, $f(4)$, $f(5)$, etc.) ♦ Interpret functional relationships in terms of a context (e.g., from a table, the graph of a line, a description, etc.)	♦ Determine whether the relationship shown in a graph represents a function (e.g., by using the vertical line test) ♦ Determine whether the relationship shown in a variety of ways represents a function (e.g., table, mapping, graph, etc.) ♦ Solve a problem involving dependent and independent variables in a real-life situation (e.g., determine the number of buses needed given the number of students going on a trip) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Functions
Interpreting Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Interpret functions that arise in applications in terms of the context (linear, quadratic, exponential, rational, polynomial, square root, cube root, trigonometric, logarithmic).	♦ Interpret positive or negative rate of change of a linear function in terms of a context (e.g., a negative rate of change in a distance/time graph indicates deceleration) ♦ Interpret mathematical relationships in terms of a context (e.g., from a table, graph, description, etc.) ♦ Create a table of ordered pairs that represents a relationship between two variables in a real-life situation (e.g., \$0.95 per donut) *See entry points for earlier grades in this domain*	♦ Interpret the rate of change of a linear function in terms of a context (e.g., in a function that represents miles traveled over time, the slope represents the average speed in miles per hour) ♦ Interpret the initial value (y-intercept) of a linear function in terms of a context (e.g., the y-intercept of a graph showing profits at a car wash represents the amount of money initially spent) ♦ Create a graph that represents the relationship between two variables in a real-life situation (e.g., the total cost of sodas at \$0.85 each)	♦ Identify the set of numbers to include as possible input values (domain) in a real-life situation (e.g., if ribbon costs a dollar per yard, the domain for the total yardage of ribbon is the set of non-negative real numbers) ♦ Identify the set of numbers to include as possible output values (range) in a real-life problem that includes real numbers (e.g., for buying eggs by the dozen, the range of values for the total number of eggs is positive multiples of 12) ♦ Interpret the rate of change over an interval of the domain on a graph of a non-linear function (e.g., for a distance/time piecewise graph determine an average rate of time between two points) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Functions
Interpreting Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Analyze functions using different representa-tions. (F-IF)	♦ Graph a linear function on a coordinate grid given a table of ordered pairs (e.g., graph the line that passes through two points) ♦ Describe a graph of a (piecewise) function that has labeled sections (e.g., part A rises gently, part B is short and flat, part C rises like part A, but is steeper, etc.) ♦ Match a graph of a function with its equation (e.g., using linear functions with different features) ♦ Match an equation of a function with a table of ordered pairs (e.g., connect the equation with the values that satisfy it) ♦ Match ordered pairs with its graph (e.g., list coordinate points or a table with ordered pairs) ♦ Identify graphs of linear, exponential, and quadratic functions (e.g., differentiate between them) *See entry points for earlier grades in this domain*	♦ Graph an exponential function on a coordinate grid given a table of ordered pairs (e.g., roughly show any intercepts, asymptote, and end behavior) ♦ Graph a quadratic function on a coordinate grid given a table of ordered pairs (e.g., show any intercepts, the vertex, and end behavior) ♦ Create a graph of a (piecewise) function based on a description (e.g., describe graphically distance traveled on a bike ride over time.) ♦ Create a table of ordered pairs based on a quadratic function (e.g., create a table for $f(x) = x^2 - 3$) ♦ Create a table of ordered pairs based on an exponential function (e.g., create a table for $f(x) = 2^x$) ♦ Compare initial values of two functions presented in different ways (e.g., for $f(x) = 3x - 5$ the initial value is -5, and for the relationship $(0,2)$, $(1,0)$, $(2,-2)$, the initial value is 2)	♦ Graph a linear, exponential, or quadratic function on a coordinate grid given its equation (e.g., graph $f(x) = -2x - 8$, $g(x) = 3^x$, etc.) ♦ Compare the rates of change for two functions presented in different ways (e.g., compare the rates of change of Ella's savings account, based on a description, and Kevin's savings account, based on an equation) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Building Functions

High School

Cluster	Standards as written	
Build a function that models a relationship between two quantities.	**H.F-BF.1**	Write a function (linear, quadratic, exponential, simple rational, radical, logarithmic, and trigonometric) that describes a relationship between two quantities.★
	H.F-BF.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.★
	H.F-BF.1b	Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ★
	H.F-BF.1c	(+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* ★
	H.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★
Build new functions from existing functions.	**H.F-BF.3**	Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x, f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.)
	H.F-BF.4	Find inverse functions algebraically and graphically.
	H.F-BF.4a	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (Include linear and simple polynomial, rational, and exponential functions.)
	H.F-BF.4b	(+) Verify by composition that one function is the inverse of another.
	H.F-BF.4c	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
	H.F-BF.4d	(+) Produce an invertible function from a non-invertible function by restricting the domain.
	H.F-BF.5	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

★ indicates Modeling standard + indicates standard is beyond College and Career Ready
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions
Building Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Build a function that models a relationship between two quantities.	♦ Determine the missing value in a table of ordered pairs that represents a linear function (e.g., a missing output value) ♦ Determine the missing value within a given arithmetic sequence by identifying an initial value and an addition/subtraction rule (e.g., 12, 7, 2, ?, -8 ...) ♦ Complete a table to extend an arithmetic sequence (e.g., complete a partial table that has some values and a linear function rule) ♦ Create an arithmetic sequence using manipulatives (e.g., add a constant number of objects) *See entry points for earlier grades in this domain*	♦ Determine the rule that defines a function based on a table of ordered pairs (e.g., the rule for values in a table with inputs 1, 2, 3, and corresponding outputs 11, 16, 21, is $5x + 6$) ♦ Determine the missing value within a given geometric sequence by identifying an multiplication/division rule (e.g., 2, 6, ?, 54, 162 ...) ♦ Determine a specific term outside of a given arithmetic sequence (e.g., determine the 12th term in the sequence)	♦ Create a sequence of ordered pairs in a table based on a function rule (e.g., for $f(x) = -3x - 11$ find $f(1)$, $f(2)$, $f(3)$. etc.) ♦ Determine a specific term outside of a given geometric sequence (e.g., determine the 8th term in the sequence) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Build new functions from existing functions.	♦ Identify the graph of a parent function (e.g., $y = x$, $y = x^2$, $y = \|x\|$, etc.) ♦ Draw the graph of a parent function (e.g., make a rough sketch of $y = x^2$) *See entry points for earlier grades in this domain*	♦ Match the graph of a function to a graph of its inverse (e.g., choose the graph that demonstrates symmetry over the line $y = x$) ♦ Graph a function over a translation of a parent graph (e.g., for $y = \|x\|$, graph $y = \|x\| - 3$)	♦ Create the equation of a translation of a parent function based on its graph (e.g., The translation of 2 units to the left of a quadratic function has the equation $y = (x + 2)^2$) ♦ Calculate the equation of the inverse of a linear function (e.g., for an equation in the form $y = mx + b$, use the equation $x = my + b$ and solve for y.) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics

CONCEPTUAL CATEGORY Functions

DOMAIN Linear, Quadratic, and Exponential Models

High School

Cluster	Standards as written	
Construct and compare linear, quadratic, and exponential models and solve problems.	**H.F-LE.1**	Distinguish between situations that can be modeled with linear functions and with exponential functions. ⋆
	H.F-LE.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ⋆
	H.F-LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ⋆
	H.F-LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ⋆
	H.F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). ⋆
	H.F-LE.3	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ⋆
	H.F-LE.4	For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ⋆
Interpret expressions for functions in terms of the situation they model.	**H.F-LE.5**	Interpret the parameters in a linear or exponential function (of the form f(x) = bx + k) in terms of a context. ⋆

⋆ indicates Modeling standard

ENTRY POINTS for Functions
Linear, Quadratic, and Exponential Models Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Construct and compare linear, quadratic, and exponential models and solve problems.	♦ Create an arithmetic sequence (e.g., a sequence of numbers with a common difference) ♦ Identify a real-life situation that can be modeled by a linear function from a table, graph, description, or symbols (e.g., the cost of hamburgers at $4 each) ♦ Construct a linear function that represents the values of ordered pairs shown in a table (e.g., for x values of 3, 4, and corresponding $f(x)$ values of 7, 10, the function is $f(x) = 3x - 2$) ♦ Distinguish between linear and non-linear models based on tables representing them (e.g., a table that does not show a constant increase represents a non-linear relationship) ♦ Distinguish between situations that are modeled by exponential growth and by exponential decay (population increase vs. a car's depreciation) *See entry points for earlier grades in this domain*	♦ Create a geometric sequence (e.g., by identifying an initial value and a multiplication/division rule) ♦ Identify a real-life situation that can be modeled by an exponential function from a table, graph, description, or symbol (e.g., the sale price of an item reduced by 10% per week) ♦ Construct an exponential function that represents the values of ordered pairs shown in a table (e.g., for x values of 1, 3, and corresponding $f(x)$ values of 2, 18, the function is $f(x) = \frac{2}{3}(3^x)$) ♦ Interpret a table that represents a real-life situation or mathematical relationship that can be modeled by a linear function, from a table, graph, description, or symbols (e.g., by finding any rate of increase/decrease and initial value) ♦ Distinguish between linear, exponential, and quadratic models (e.g., from a table, equation, graph, description, etc.)	♦ Distinguish between a linear model and a non-linear model based on a real-life situation (e.g., bank account growth from deposits vs. growth from interest) ♦ Distinguish between exponential and non-exponential models based on tables representing them (e.g., a table that does not show a constant increase represents a non-linear relationship) ♦ Interpret a table that represents a real-life situation or mathematical relationship that can be modeled by an exponential function, from a table, graph, description, or symbol (e.g., by finding any growth/decay rate and initial value) ♦ Compare a linear, an exponential, and a quadratic function for several inputs (e.g., to illustrate that exponential growth will eventually exceed linear and quadratic growth) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Interpret expressions for functions in terms of the situation they model. (continued)	♦ Identify the parameters of a linear function (e.g., for the function $f(x) = mx + b$, m represents the rate of change and b represents the initial value) ♦ Identify the parameters of an exponential function (e.g., for the function $f(x) = a(b)^x$, a represents the initial value, b represents the growth/decay factor, and x represents the independent variable) *See entry points for earlier grades in this domain*	♦ Identify the parameters of a linear function that models a real-life situation (e.g., $f(x) = 8.5x - 25$ represents profits from a car wash; \$8.50 represents the amount charged per car wash and \$25 represents the cost of materials) ♦ Compare the parameters of 2 or more linear functions from a table or a description (e.g., which has the greater rate of change and/or initial value) ♦ Compare the parameters of 2 or more linear functions that represent real-life situations from a table or a description (e.g., which has the greater rate of change and/or initial value)	♦ Identify the parameters of an exponential function that models a real-life situation (e.g., $f(x) = 100(1.02)^6$ represents Leo's bank account; \$100 is the initial value, 2% is the growth rate, 6 is the number of years) ♦ Compare the parameters of 2 or more exponential functions from a table or a description (e.g., which has the greater growth/decay rate and/or initial value) ♦ Compare the parameters of 2 or more exponential functions that represent real-life situations from a table or a description (e.g., which has the greater growth/decay rate and/or initial value) ♦ Compare the parameters of 2 or more functions of different types that represent real-life situations from a table or a description (e.g., which has the greater initial value and/or which has the greater value over an interval of the independent variable between a linear and an exponential model) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics

CONCEPTUAL CATEGORY Functions

DOMAIN Trigonometric Functions

High School

Cluster	Standards as written	
Extend the domain of trigonometric functions using the unit circle.	**H.F-TF.1**	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
	H.F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
	H.F-TF.3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
	H.F-TF.4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions.	**H.F-TF.5**	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.⋆
	H.F-TF.6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
	H.F-TF.7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. ⋆
Prove and apply trigonometric identities.	**H.F-TF.8**	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant.
	H.F-TF.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

⋆ indicates Modeling standard

+ indicates standard is beyond College and Career Ready

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Extend the domain of trigonometric functions using the unit circle.	These standards are not common to both high school pathways and will not be assessed.		
Model periodic phenomena with trigonometric functions.	These standards are not common to both high school pathways and will not be assessed.		
Prove and apply trigonometric identities.	These standards are not common to both high school pathways and will not be assessed.		